

Evaluation of Selected Value-at-Risk Approaches in Normal and Extreme Market Conditions

Dissertation submitted in part fulfilment of the
requirements for the degree of Master of Science in
International Accounting and Finance

at Dublin Business School



August 2014

Submitted and written by

Felix Goldbrunner

1737701

Declaration:

I declare that all the work in this dissertation is entirely my own unless the words have been placed in inverted commas and referenced with the original source. Furthermore, texts cited are referenced as such, and placed in the reference section. A full reference section is included within this thesis.

No part of this work has been previously submitted for assessment, in any form, either at Dublin Business School or any other institution.

Signed:.....

Date:.....

Table of Contents

Table of Contents	4
List of Figures	6
List of Tables.....	7
Acknowledgements	8
Abstract	9
1. Introduction	10
1.1 Aims and Rationale for the Proposed Research	10
1.2 Recipients for Research.....	10
1.3 New and Relevant Research.....	10
1.4 Suitability of Researcher for the Research	11
1.5 General Definition.....	11
2. Literature Review	14
2.1 Theory	14
2.1.1 Non-Parametric Approaches	14
2.1.2 Parametric Approaches	15
2.1.3 Simulation – Approach.....	29
2.2 Empirical Studies	31
2.2.1 Historical Simulation.....	31
2.2.2 GARCH.....	32
2.2.3 RiskMetrics	32
2.2.4 IGARCH.....	33
2.2.5 FIGARCH	34
2.2.6 GJR-GARCH	34
2.2.7 APARCH.....	35
2.2.8 EGARCH	36
2.2.9 Monte Carlo Simulation	36
3. Research Methodology and Methods	37
3.1 Research Hypotheses.....	37
3.2 Research Philosophy	39
3.3 Research Strategy.....	39
3.4 Ethical Issues and Procedure.....	45
Research Ethics	45
3.5 Population and Sample.....	45
3.6 Data Collection, Editing, Coding and Analysis	49

4. Data Analysis	50
4.1 Analysis of the period from 2003 to 2013.....	51
4.2 Analysis of the period from 2003 to 2007.....	54
4.3 Analysis of the period from 2008 to 2013.....	58
5. Discussion	61
5.1 Discussion	61
5.2 Research Limitations and Constrains	63
6. Conclusion.....	64
Publication bibliography	66
Appendix A: Reflections on Learning	73
Appendix B.I Oxmetrics Output Crisis Sample	75
Appendix B.II Oxmetrics Output Pre-Crisis Sample	94
Appendix B.III Oxmetrics Output Full Sample	114
Appendix C Oxmetrics Screenshots.....	133

List of Figures

Figure 1: Distribution and Quantile.....	12
Figure 2: Daily Returns (CDAX)	13
Figure 3: Histogram of Daily Returns (CDAX)	13
Figure 4: Volatility Overview (CDAX)	16
Figure 5: Correlogram of Squared Returns (CDAX): 1 year	17
Figure 6: Absolute Returns (CDAX).....	27
Figure 7: Correlogram of Absolute Returns (CDAX); 2003-2014.....	27
Figure 8: Autocorrelation of Returns to the Power of d.....	28
Figure 9: LR(uc) and Violations.....	41
Figure 10: Violation Clustering.....	43
Figure 11: Price Chart	46
Figure 12: Return Series.....	47
Figure 13: Histogram, Density Fit and QQ-Plot	47
Figure 14: VaR Intersections.....	63

List of Tables

Table 1: Non-rejection Intervals for Number of Violations x 42

Table 2: Conditional Exceptions 43

Table 3: Descriptive Statistics 48

Table 4: Descriptive Statistics Sub-Samples 49

Table 5: Ranking (2003-2013) 51

Table 6: Test Statistics (2003-2013)..... 53

Table 7: Ranking (2003-2007) 55

Table 8: Test Statistics (2003-2007)..... 57

Table 9: Ranking (2008-2013) 58

Table 10: Test Statistics (2008-2013)..... 60

Table 11: Ranking Overview..... 61

Acknowledgements

I would like to thank my family for their support during the last year, without their support the completion of this program and thesis would not have been possible. Also I wish to express my gratitude to my supervisor Mr. Andrew Quinn, without his support and creative impulses this thesis would not be as it is today.

Abstract

This thesis aimed to identify the approaches with the most academic impact and to explain them in greater detail. Hence, models of each category were chosen and compared. The non-parametric models were represented by the historical simulation, the parametric models by GARCH-type models (GARCH, RiskMetrics, IGARCH, FIGARCH, GJR, APARCH and EGARCH) and the semi-parametric models by the Monte Carlo simulation. The functional principle of each approach was explained, compared and contrasted.

Test for conditional and unconditional coverage were then applied to these models and revealed that models accounting for asymmetry and long memory predicted value-at-risk with sufficient accuracy. Basis for this were daily returns of the German CDAX from 2003 to 2013.

1. Introduction

1.1 Aims and Rationale for the Proposed Research

Recalling the disastrous consequences of the financial crisis, it becomes apparent that the risks taken by financial institutions can have significant influences on the real economy. The management of these risks is therefore essential for the functioning of financial markets and consequently for the performance of the whole economy. Legislators and regulators have therefore set their focus on various risk-management frameworks and even derived capital requirements in accordance with certain risk measures. The most prominent of these is the so called value at risk (VaR) measure, which was developed by J.P. Morgan at the end of the 80s and tries to identify *the worst loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger* (Jorion 2007b).

Value at risk plays an important role in the risk management of financial institutions. Its accuracy and viability, both in normal and more extreme economic climates, is therefore desirable. Since its introduction, academics and practitioners have developed a vast number of methods to determine VaR, all of which are based on different assumptions and perspectives. The question of finding an approach that delivers accurate results in normal and extreme market conditions therefore poses a problem.

The aim of this thesis is to solve this problem and to answer the question concerning the most accurate approach to determine value at risk in both normal and more extreme market conditions.

1.2 Recipients for Research

The main recipients of this research will be managers responsible for risk management in financial institutions such as banks and hedge funds as well as other financial-service providers. Since this thesis aims also to explain the various value at risk approaches in a generally intelligible way, independent and less-sophisticated investors can also be numbered among the recipients. Additionally, researchers in the academic area of risk management, who developed the models that will be tested, will also be beneficiaries of this research.

1.3 New and Relevant Research

To analyze the various approaches to value at risk, this thesis will identify the most accurate approaches according to literature and then test them in terms of accuracy under both normal market conditions and crisis conditions. In this way, a ranking will be proposed which will show the most suitable methods for calculating value at risk. Most especially, the comparison between normal function and function in a time of crisis is new and relevant research which has not been thoroughly discussed in previous literature. As a result, practitioners as well as academic researchers can benefit from this research.

1.4 Suitability of Researcher for the Research

To conduct this research, the researcher needs to be confident in approaching and utilizing both fundamental and advanced statistics. Deeper knowledge about capital markets is required as well as an understanding of widely used risk-management techniques. Moreover, the researcher should be experienced in working with current spreadsheet applications such as Microsoft Excel© or numerical computing suits such as OxMetrics©. The researcher has the required experience in all of these areas, evidenced through his undergraduate degree (Upper Second Class Honours in BSc in Business Administration) at the Catholic University of Eichstätt-Ingolstadt, where he has already conducted research on the new liquidity requirements proposed in the new Basel III regulation and on contingent capital with regard to its contribution to the stability of financial markets.

Moreover, this researcher's current Master's-level course in international accounting and finance enhances his knowledge of risk management and capital markets. My working experience in form of a bank internship will also facilitate my perspective on the chosen topic.

1.5 General Definition

Value at risk is risk metric which measures the market risk in the future value of an asset or portfolio. It is therefore a measure of uncertainty of a portfolio's profit and loss (P&L), i.e., returns. To measure this risk, the portfolio's profit and loss deviations from an expected value are needed. This factor is called volatility and is the standard deviation σ from an expected value μ . When considering a portfolio of assets, the correlation of the assets within the portfolio is also a critical factor. To derive all these factors, assumptions have to be made about the assets profit and/or loss distribution (Alexander 2009).

Combining all these risk factors, the value at risk can be defined as:

Definition 1:

The worst loss over a target horizon such that there is a low, prespecified probability (confidence level) that the actual loss will be larger (Jorion 2007a).

It is therefore possible to come to statements of the following form:

“With a certainty of X percent, the portfolio will not lose more than V dollars over the time T .”

Mathematically, this is the pre-specified upper bound of the loss distribution, the $1-\alpha$ quantile (Emmer et al. 2013) :

$$VaR_{\alpha}(L) = q_{\alpha}(L) = \inf\{\ell | Pr(L \leq \ell) \geq \alpha\} \quad (1.1)$$

where :

$L = Loss$

or when considering the whole P&L distribution the pre-specified lower bound, the α quantile (Acerbi & Tasche 2002):

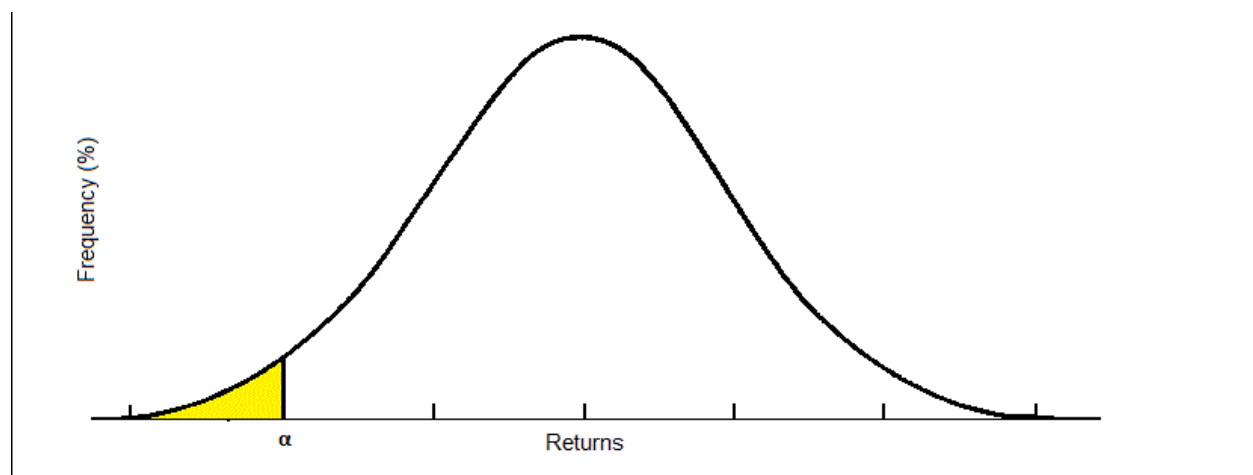
$$VaR_{\alpha}(X) = q_{\alpha}(X) = \sup\{x | Pr(X \leq x) \leq \alpha\} \quad (1.2)$$

where:

X
 = Random variable describing the future value of profit or loss; Returns

To illustrate this, Figure 1 depicts the distribution of asset returns and highlights the alpha quantile.

Figure 1: Distribution and Quantile



To measure these returns, there are two possibilities: the arithmetic and the geometric rate of return.

The arithmetic returns are comprised by the capital gain $P_t - P_{t-1}$ plus interim payments D_t and can be defined as follows (Jorion 2007a):

$$u_i = \frac{P_t + D_t - P_{t-1}}{P_{t-1}} \quad (1.3)$$

Where:

P_t = Asset price at time t

P_{t-1} = Previous asset price

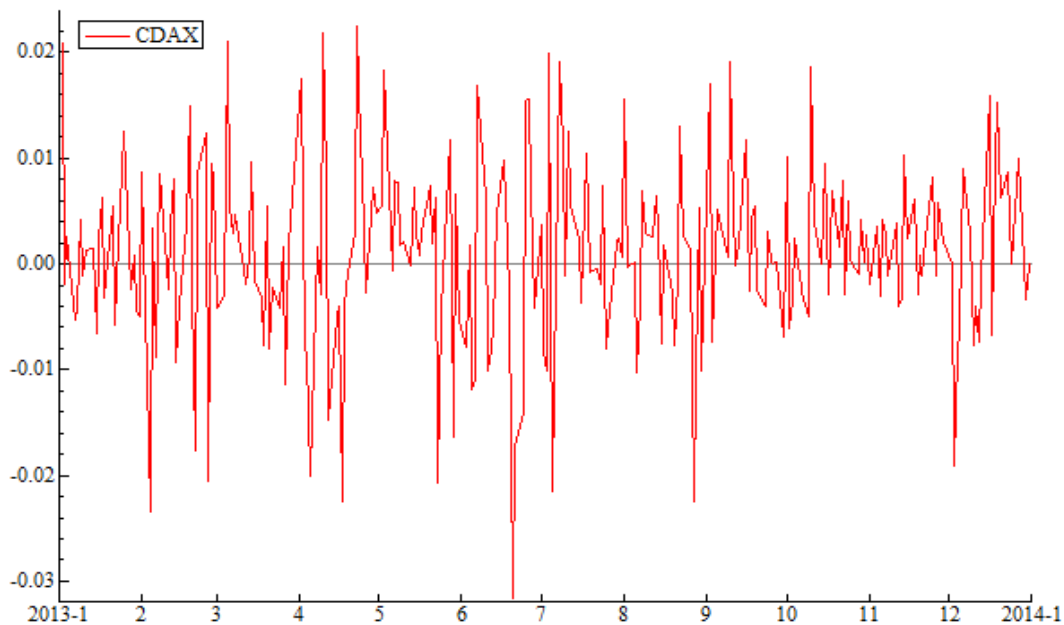
D_t = Interim payments, such as dividends or coupons

Instead of this measurement, geometric returns also seem natural. These returns are expressed in terms of the logarithmic price ratio, which has the advantage of not leading to negative prices, as is mathematically possible with arithmetic returns:

$$u_i = \ln\left(\frac{P_t + D_t}{P_{t-1}}\right) \quad (1.4)$$

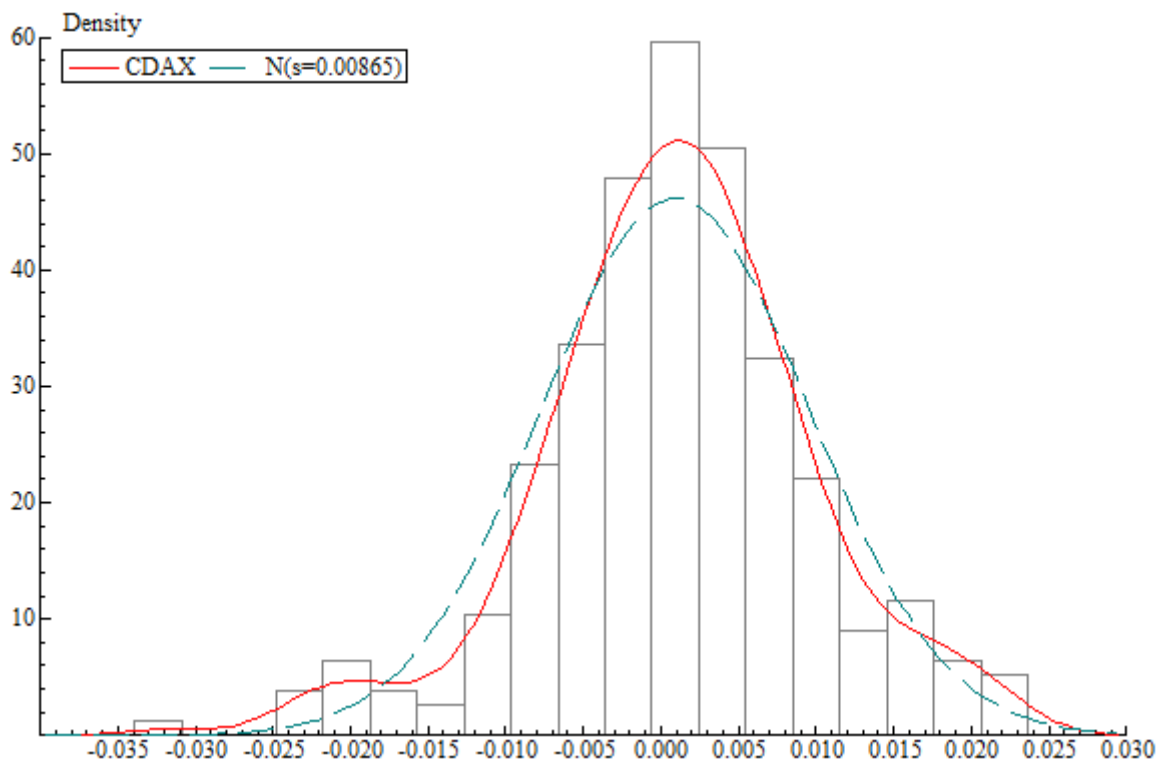
Figure 2 shows an example of logarithmic returns for the German Composite DAX (CDAX).

Figure 2: Daily Returns (CDAX)



When summing up these returns, another advantage is apparent. Other than the arithmetic method, the geometric return of two periods is simply the sum of the individual returns. These returns are sorted according to size and their frequency and result in the sampling distribution, which is in practice often assumed to be a normal distribution.

Figure 3: Histogram of Daily Returns (CDAX)



To illustrate this historical distribution, a graphical analysis can be conducted in the form of a histogram and the samples density function, as shown in Figure 3. It can be seen that the

sample distribution (red line) is somewhat similar to a normal distribution (green line). Most especially, the tails (quantiles) differ significantly from the normal distribution in this example.

For this reason, various approaches have been developed to estimate value at risk to cope with these observations. These will be introduced in section 2.1 and the respective literature reviewed in 2.2. Section three will then outline the research methodology and describe the selected sample. Section four will present the findings of the research conducted and a discussion will be provided in section five. Finally, section six will summarize the findings and conclude.

2. Literature Review

The literature review will be organized as follows: First, the theory behind the selected approaches will be explained. The second part will then consist of recent empirical studies incorporating the explained approaches.

2.1 Theory

Manganelli & Engle (2001) classify existing value at risk approaches in three categories with regard to their parameterization of stock price behavior: parametric, non-parametric, and semi-parametric. These categories will be maintained for the analysis of approaches in the thesis. At least one approach per category will be tested. Additionally, said categories will also be maintained in this literature review.

2.1.1 Non-Parametric Approaches

According to Powell & Yun Hsing Cheung (2012), it is unlikely that stock returns follow a parametric distribution, especially in times of a financial crisis. They therefore suggest the use of non-parametric calculation methods, which will now be considered in more detail.

2.1.1.1 Historical Simulation

The most prominent and also the most straightforward method to calculate value at risk is historical simulation.

Its simplicity lies in that no assumption about the population has to be made, since the actual historical probability density function is used to derive VaR. The key assumption is therefore that history repeats itself. A possible scenario in this framework is that an asset's change in market value from today to tomorrow will match an actual observed change that occurred between a consecutive pair of dates in the past (Picoult 2002)

To calculate VaR, the n^{th} lowest observation has to be found, where n equals α of the corresponding confidence level $(1 - \alpha)$ of the value at risk. In other words, the α -quantile has to be determined as defined in (1.2). (Powell & Yun Hsing Cheung 2012).

Given a sample of 252 trading days, which equals about one year, the 95th percentile ($\alpha=0,05$) would be the 13th largest loss or lowest return. From this, it also follows that the 99th percentile will not be a constant multiple of the 95th percentile, and vice versa. Moreover, a 10-day VaR will not be a constant multiple of the one-day VaR. These limitations are a result of not assuming independent and identically distributed (IID) random variables (Hendricks 1996).

2.1.2 Parametric Approaches

Parametric approaches try to simplify the calculation by making assumptions about the underlying probability distribution. Parameters are estimated and eventually used to derive value at risk.

In case of a normal distribution (1.2) is then simply (Lee & Su 2012):

$$VaR = \mu + F(u_t) \cdot \sigma_t \quad (2.1)$$

where:

μ : mean of population

σ_t : volatility

$F(u_t)$: left tailed quantile i. e. inverse of the cumulative normal distributio

When choosing another distribution $F(u_t)$ simply represents the left-tailed quantile of the selected distribution.

It can be seen that the estimation of volatility is a critical factor to determine. To do this, several approaches are available, and each of these which will be explained in the next part after giving some essential definitions.

The most important factor in this context is volatility:

Definition 2:

Volatility σ is the standard deviation of the returns of a variable per time unit (Hull 2012). It is therefore a measure dispersion.

Linked to this is the measure variance σ^2 , which is simply the squared standard deviation.

Mathematically, the variance and consequently the standard deviation of a sample are estimated as:

$$\sigma_n^2 = \frac{1}{m-1} \sum_{i=1}^m (u_{n-1} - \bar{u})^2 \quad (2.2)$$

where:

m = Number of observations

\bar{u} = Mean of u_i

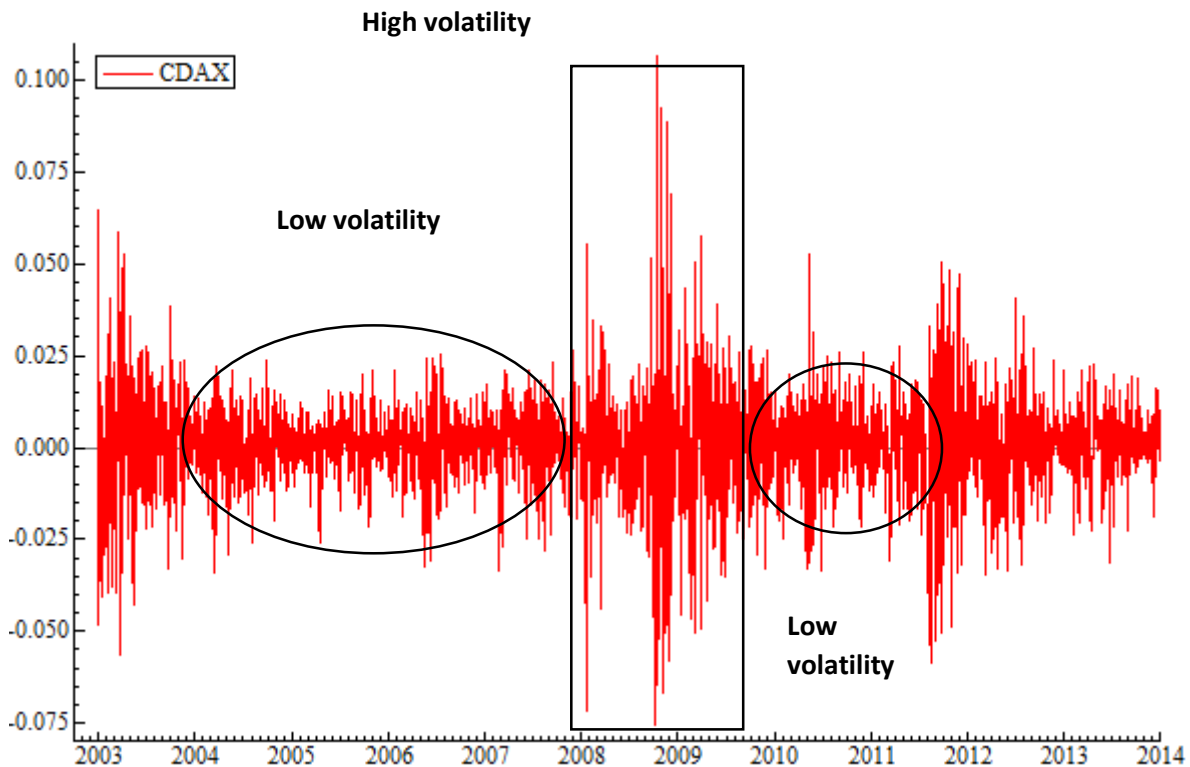
Assuming that the sample fully represents the population and that this population follows a normal distribution with the mean $\bar{u} = 0$ allows us to reduce (2.2) to the following form:

$$\sigma_n^2 = \frac{1}{m} \sum_{i=1}^m u_{n-1}^2 \quad (2.3)$$

Note that this form of variance is also known as unweighted variance, as it weighs each return equally. When looking of the daily returns of the CDAX for the last ten years (Figure 4) another observation related to this can be made. When the index price made a significant move on one day, the occurrence of a significant change the following day was more likely. It can be seen that volatility increased dramatically during the financial crisis of 2008, but then

eventually returned to approximately the same level as before the crisis, to increase again with the beginning of the Euro crisis.

Figure 4: Volatility Overview (CDAX)



These phenomena are called heteroscedasticity and autocorrelation and are the main reason for volatility clustering:

Definition 3:

Heteroscedasticity is the property of random variables that sub-populations have a different variances. Variance is therefore not constant. The opposite is called: homoscedastic

Definition 4:

Autocorrelation is a correlation of the values of a variable with values of the same variable lagged one or more time periods back (Aczel & Sounderpandian 2009).

Similar to the principals of covariance and correlation, the autocovariance (2.4) and autocorrelation (2.5) can be defined as follows:

$$\hat{\gamma}_j = \frac{1}{T-j-1} \sum_{t=j+1}^T (u_t - \bar{u})(u_{t-j} - \bar{u}) \quad (2.4)$$

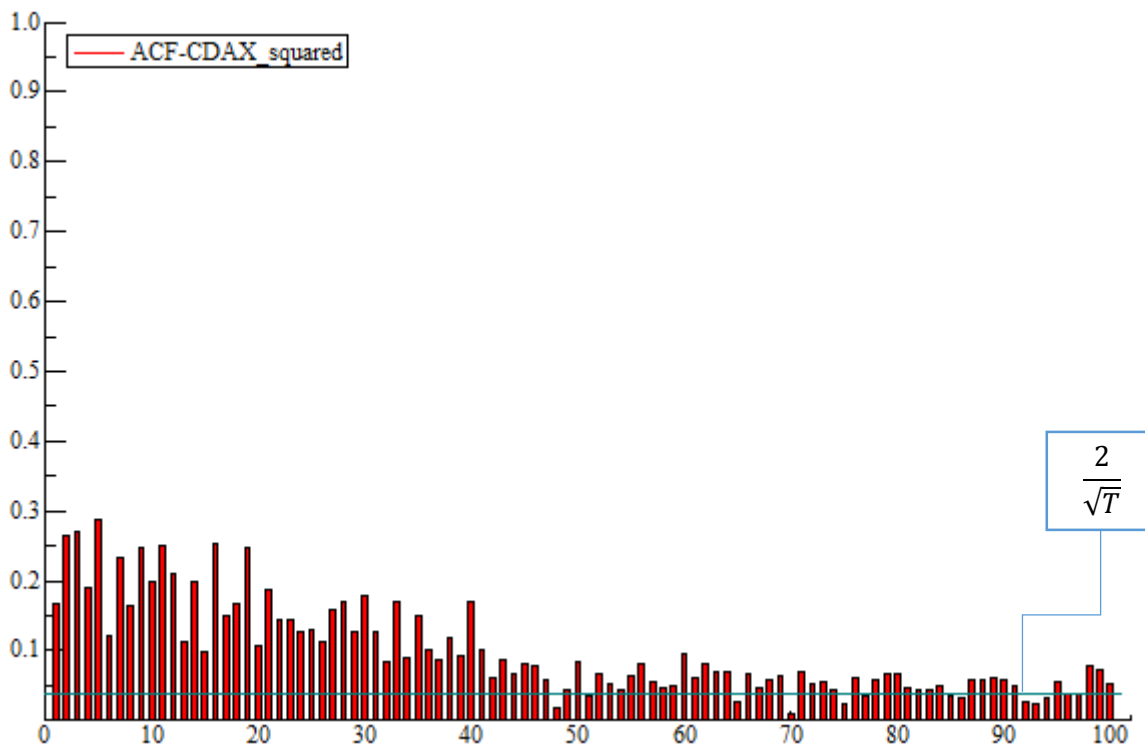
where:

j = Number of lags

$$\hat{\rho}_j = r_j := \frac{\sum_{t=j+1}^T (u_t - \bar{u})(u_{t-j} - \bar{u})}{\sum_{t=1}^T (u_t - \bar{u})^2} \quad (2.5)$$

A graphical analysis can then be conducted in the form of a correlogram as exemplified by Figure 5, which plots r_j against the number of lags for the squared CDAX returns from 2003 to 2013:

Figure 5: Correlogram of Squared Returns (CDAX): 1 year



It can be seen that the squared returns are positively autocorrelated. Their autocorrelation function (ACF) starts at approximately 0.16 and peaks at 0.3 at lag 5. After some smaller peaks for greater lags, the ACF then decreases slowly.

To test whether the autocorrelation coefficients are significantly different from zero, a two-sided significance test of normal distribution can be applied, since the coefficients should be nearly normally distributed (Harvey 1993; Cummins et al. 2014)¹.

The null hypothesis here is then:

$$H_0: \rho_j = 0 \quad (2.6)$$

¹ r_j is considered a realization of the random variables u_t

At a significance level of $\alpha = 0.05$, H_0 will be rejected when:

$$|r_j| > \frac{2}{\sqrt{T}} \quad (2.7)$$

This is represented by the blue line in Figure 5.

These findings might imply that the observed variables, returns, are not IID. Keeping this in mind, (2.2) and (2.3) reflect only a distorted image of variance. The estimation thus needs adjustment.

It becomes apparent that, in the presence of autocorrelation, it might be useful to apply weights to the observed returns when estimating volatility. The more recent observations shall therefore be given more weight than older ones, as they have more influence on today's or tomorrow's volatility than older ones. (2.3) thus transforms to (Hull 2011):

$$\sigma_n^2 = \sum_{i=1}^m \alpha_i u_{n-i}^2 \quad (2.8)$$

where:

$\alpha_i = \text{weight for the observation } i \text{ days ago } (> 0)$

restrained by the following:

$$\sum_{i=1}^m \alpha_i = 1 \quad (2.9)$$

and:

$$\alpha_i < \alpha_j \text{ for } i > j \quad (2.10)$$

(2.7) will then give greater weight to more recent observations.

2.1.2.1 ARCH (q) - Method

Taking the fact into account that volatility decreased again after its peaks in 2008 and 2003 to a “pre-crisis” level, a further adjustment might be needed. When assuming that a long-term average variance V_L exists², (2.8) can be extended to:

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^m \alpha_i u_{n-1}^2 \quad (2.11)$$

where:

V_L = long – term average variance

γ = weight assigned to V_L

(2.9) then changes accordingly to:

$$\gamma + \sum_{i=1}^m \alpha_i = 1 \quad (2.12)$$

This model of weighing the observations and additionally considering a long-term element is called the univariate autoregressive conditional heteroscedasticity (ARCH) model and was first developed by Engle (1982) and applied to the inflation rate of the United Kingdom.

2.1.2.2 GARCH (p,q) - Method

Bollerslev (1986) then generalized the model to the so-called **Generalized Autoregressive Conditional Heteroscedasticity** (GARCH) model. The difference is that, next to the long-term average volatility V_L and the recent squared returns, the recent variance can also be taken into account.

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (2.13)$$

With $\gamma + \alpha + \beta = 1$ begin the weights of: (2.14)

V_L : long term average variance

u_{n-1} : previous daily return

σ_{n-1}^2 : previous variance

An important restriction next to (2.14) to ensure the positivity of σ_n^2 is that:

$$\omega > 0 \quad (2.15a)$$

$$\alpha \geq 0 \quad (2.15b)$$

$$\beta \geq 0 \quad (2.15c)$$

In case $\alpha = 0$, β also has to be set to zero, as otherwise this would lead to a constant σ_n^2 with β being unidentifiable.

² Also called unconditional variance; σ_n^2 is equivalently called conditional variance

In terms of notation, the GARCH model is often denominated as GARCH (p,q), where p is the lag for the recent variance $\beta\sigma_{n-p}^2$ and q the lag for the squared returns αu_{n-q}^2 . A GARCH (1,1) model accounts thus for one lag each.

When defining $\omega = \gamma V_L$, (2.13) can be rewritten as:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (2.16)$$

The parameters ω , α , and β can then to be estimated by a quasi-maximum likelihood (QML) estimation.

Using a lag operator, the model can also be expressed as follows:

$$\sigma_n^2 = \omega + \alpha(L)u_t^2 + \beta(L)\sigma_t^2 \quad (2.17)$$

where:

$$\begin{aligned} \alpha(L) &= \alpha_1 L + \alpha_2 L^2 + \dots + \alpha_q L^q \\ \beta(L) &= \beta_1 L + \beta_2 L^2 + \dots + \beta_p L^p \end{aligned}$$

Given (2.14), γ is simply $\gamma = 1 - \alpha - \beta$ and V_L can be defined as

$$V_L = \frac{\omega}{\gamma} = \frac{\omega}{1 - \alpha - \beta} \quad (2.18)$$

To give an explanation for how this model considers autocorrelation, the correlation coefficient ρ_j has to be reformulated (Bauwens et al. 2012) together with

$$\rho_1 = \frac{\alpha(1 - \beta^2 - \alpha\beta)}{(1 - \beta^2 - 2\alpha\beta)} \quad (2.19)$$

where $\rho_1 > \alpha$

and

$$\rho_j = \frac{(\alpha + \beta)}{\rho_{j-1}} \quad (2.20)$$

For $j \geq 2$ if $\alpha + \beta < 1$

where the restriction ensures that (2.18) exists. Furthermore, it is important to note that $(\alpha + \beta)$ can also be seen as the decay factor of the autocorrelations.

As stated earlier, the parameters have to be estimated with the help of a maximum likelihood estimation. This is done by maximization of the log-likelihood function of the underlying distribution.

In the case of a normal distribution, it is assumed that

$$u_t = z_t \sigma_t \quad (2.21)$$

where:

σ_t : time varying volatility

z_t : iid $\sim N(0,1)$

The log-likelihood function is defined as (Jorion 2007a):

$$\max F(\omega, \alpha, \beta | u) = \sum_{t=1}^T \left(\ln \frac{1}{\sqrt{2\pi\sigma_t^2}} - \frac{u_t^2}{2\sigma_t^2} \right) \quad (2.22)$$

The values of ω , α , and β , that maximize this function can then be used in the GARCH model.

Looking at Figure 3, it can be seen that the normal distribution is not always the most applicable assumption. In fact, Mandelbrot (1963) and Fama (1965) find fatter and longer tails than normal distribution and consequently ask for other more suitable distributions. Similarly, Kearns & Pagan (1997) observe that returns are actually not IID. Following these findings, Praetz (1972), Bollerslev (1987), Baillie & DeGennaro (1990), Mohammad & Ansari, (2013) and others suggests the application of the student's t-distribution for (2.21). (2.22) thus changes to (Bauwens et al. 2012)

$$\max F(\omega, \alpha, \beta | u, v) \quad (2.23)$$

$$= \sum_{t=1}^T \left(\ln \left[\frac{\Gamma\left(v + \frac{1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \right] - \frac{1}{2} \left[(v-2)\sigma_t^2 + \frac{(v+1)(u_t - \mu_t)^2}{(v-2)\sigma_t^2} \right] \right)$$

with $v > 2$

(2.24)

where:

$\Gamma(\cdot)$: Gamma function

v : Degree of freedom

Similarly, it is also possible to adjust (2.22) or (2.23) for the skewedness in form of a skewed student t-distribution. This is then (Lambert & Laurent 2001):

$$\max F(\omega, \alpha, \beta | u, v, \xi) \quad (2.25)$$

$$= \sum_{t=1}^T \left(\ln \left[\frac{\Gamma\left(v + \frac{1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \right] \right. \\ \left. - \frac{1}{2} \ln[\pi(v-2)] \right. \\ \left. + \ln \left(\frac{2}{\xi + \frac{1}{\xi}} \right) \right. \\ \left. + \ln(\sigma) - \frac{1}{2} \sum_{t=1}^T \left[\ln \sigma_t^2 + (1+v) \ln \left(1 + \frac{\sigma u_t + \mu}{v-2} \xi^{-l_t} \right) \right] \right)$$

$$\text{with } v > 2 \quad (2.26)$$

where:

$\Gamma(\cdot)$: Gamma function

v : Degree of freedom

ξ : Skewness parameter

This is then also transferable to other models.

Before moving on to the next model of estimating volatility, the major limitations of the GARCH model should be noted (Nelson 1991).

Firstly, the GARCH model is limited by the constraints given in (2.15a), (2.15b) and (2.15c). As stated by Nelson & Cao (1992), Bentes et al. (2013a) and Nelson (1991) the estimation of the parameters in fact often violates these constraints and thus restricts the dynamics of σ_n^2 .

Secondly, the GARCH methods models shock persistence according to an autoregressive moving average (ARMA) of the squared returns. According to Lamoureux & Lastrapes (1990) this does not concur with empirical findings and often leads to an overestimation of persistence with the growing frequency of observations compared to other models, as stated by Carnero et al. (2004). Hamilton & Susmel (1994) see the reason for this in the fact that extreme shocks have their origin in different causes and thus also have deviating consequences for the volatility following this shock.

Finally, another significant drawback worth mentioning is that GARCH ignores by assumption the fact that negative shocks have a greater impact on subsequent volatility than positive ones. The reasons for this are unclear, but might be the result of leverage in companies according to Black (1976) and Christie (1982). A GARCH model, however, assumes that only the extent of the underlying returns determines volatility (Nelson 1991) and not the nature of the movements.

2.1.2.3 RiskMetrics - Method

Since value at risk was developed by J.P. Morgan and later distributed by the spin-off company RiskMetrics, it appears reasonable to compare the GARCH (p,q) to this approach.

The core of this model is again the measurement of volatility. To do this, the RiskMetrics approach uses an exponentially weighted moving average (EWMA) to determine volatility. The formula has the following form (J.P. Morgan 1996):

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1 - \lambda)u_{n-1}^2 \quad (2.27)$$

For $j \geq 2$ if $\alpha + \beta < 1$

When σ_{n-1}^2 is substituted, (2.26) changes to:

$$\sigma_n^2 = (1 - \lambda)(u_{n-1}^2 + \lambda u_{n-2}^2) + \lambda^2 \sigma_{n-2}^2 \quad (2.27')$$

Continuing with substituting σ_{n-2}^2 and subsequently σ_{n-3}^2 leads then to (Hull 2012):

$$\sigma_n^2 = (1 - \lambda) \sum_{i=1}^m \lambda^{i-1} u_{n-i}^2 + \lambda^m \sigma_{n-m}^2 \quad (2.28)$$

By doing this, it can be seen why this method is called the exponentially weighted moving average. The closer u^2 , the more weight is given to it, and vice versa. The rate at which the weighting is applied is λ . In the original model by RiskMetrics, λ is estimated with $\lambda = 0.94$.

It can be seen that the EWMA approach is just a special case of the GARCH model without mean reversion to the long-run variance and $\alpha = 1 - \lambda$ and respectively $\beta = \lambda$.

2.1.2.4 IGARCH (p,q) - Method

As explained in the limitations of the GARCH model, one limitation that it does not account for shock persistence, i.e., its memory. To adjust the model for this, a closer look at the expressions (2.19) and (2.20) will be taken. As explained, $(\alpha + \beta)$ is considered a decay factor of the autocorrelations; it is thus a measure p for the persistence of shocks to volatility (Carnero et al. 2004)

$$p = (\alpha + \beta) \quad (2.29)$$

Recalling (2.17), the equation can be changed to:

$$V_L = \frac{\omega}{1 - p} \quad (2.18')$$

This shows that, if $p < 1$, as secured by the constraints, u_t has a finite unconditional variance. Multi-step forecast of variance would thus approach the unconditional variance (Engle & Bollerslev 1986).

Similarly, (2.18) can be changed to (Carnero et al. 2004) :

$$\rho_1(h) = \begin{cases} \frac{\alpha(1 - p^2 - p\alpha)}{(1 - p^2 - \alpha^2)} & , h = 1 \\ \rho_1(1)p^{h-1}, & h > 1 \end{cases} \quad (2.19')$$

From this, it follows that the autocorrelation of u_t^2 reduces exponentially to zero with parameter p . (2.18') also depicts the relationship between α and $\rho_1(h)$, since α measures the dependence between squared returns for a given persistence and, therefore, of successive autocorrelations³. According to Carnero et al. (2004), this is why α plays the most important part in volatility dynamics.

Empirically, the memory property can be best described by an autocorrelogram. As shown in Figure 5, the autocorrelation of squared returns relatively high and decays slowly. Not until a lag of 47 does the autocorrelation drop below the significance level and even after that it continues to be significantly positive for some further lags. This property is called long memory.

Definition 5:

The long memory property of a financial time series is the observation of significant positive serial correlation over long lags (Ding et al. 1993)

To account for the high persistence found empirically, Engle & Bollerslev (1986) suggest setting $p = 1$ as a new constraint.

The GARCH equation then changes to:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + (1 - \alpha)\sigma_{n-1}^2 \quad (2.30)$$

with:

$$p = \alpha + \beta = 1 \quad (2.31)$$

It is obvious that, in this case, the unconditional variance and ω do not exist, since $p < 1$ would be necessary for this (see (2.18')).

Adjusting (2.31) leads to the formula below, which is also known as the Integrated GARCH (IGARCH) model without trend:

$$\sigma_n^2 = \alpha u_{n-1}^2 + (1 - \alpha)\sigma_{n-1}^2 \quad (2.32)$$

with:

$$p = \alpha + \beta = 1 \quad (2.33)$$

Allowing for a trend would then again equal (2.30) with the trend ω , but without the restriction (2.14).

Taking a closer look at (2.31) and (2.32), it can be clearly seen that the RiskMetrics approach is closely related to the IGARCH model and is in fact a special case of IGARCH, namely when $\mu = 0$.

³ $\rho_1(h)$ increases with α

Using a lag-operator again, this allows us to reformulate (2.30) to (Laurent 2014):

$$\phi(L)(1-L)u_t^2 = \omega + [1 - \beta(L)](u_t^2 - \sigma_t^2) \quad (2.34)$$

with:

$$\phi(L) = [1 - \alpha(L) - \beta(L)](1-L) \text{ of order } \max\{p,q\} - 1$$

Rearranging and adjusting then leads to the following form (Laurent 2014):

$$\sigma_t^2 = \frac{\omega}{[1 - \beta(L)]} + \{1 - \phi(L)(1-L)[1 - \beta(L)]^{-1}\}u_t^2 \quad (2.35)$$

with:

$$\phi(L) = [1 - \alpha(L) - \beta(L)](1-L) \text{ of order } \max\{p,q\} - 1$$

2.1.2.5 FIGARCH (p,d,q)

By introducing the constraint (2.33), the IGARCH model does account for high shock persistence. This, however, leads to the very restrictive assumption of infinite persistence of a volatility shock (Baillie & Morana 2009). This might limit the model, since in practice it is often found that volatility is mean reverting.

To answer for this observation, Baillie et al. (1996), suggested a fractionally integrated generalized autoregressive conditional heteroscedasticity model (FIGARCH). This model is obtained by adjusting Formula (2.35) of the IGARCH model. The first difference operator $(1-L)$ was consequently replaced with the fractional differencing operator $(1-L)^d$, where d is a fraction (Tayefi & Ramanathan 2012).

The conditional variance is then given by:

$$\sigma_t^2 = \omega[1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1}\phi(L)(1-L)^d\}u_t^2 \quad (2.36)$$

with:

$$0 < d < 1$$

2.1.2.6 GJR-GARCH

Motivated by the research of Black (1976), Christie (1982), and Nelson (1991), many models have been developed to incorporate the asymmetric effects of news on volatility, which means that different kind of shocks have different consequences on today's or on future volatility.

This phenomenon is also known as:

Definition 6:

Leverage effect denotes the fact that volatility tend to increase more as a consequence of negative return shocks than positive ones. (Bauwens et al. 2012).

One of the models which accounts for this effect is the GJR-GARCH, or simply GJR, model, which is named after its developers: Lawrence Glosten, Ravi Jagannathan, and David Runkle.

Glosten et al. (1993) give an alternative explanation of the leverage effect by noting that, when discount rates are assumed to be constant, a unanticipated decrease in future cash flows decreases the price of the asset. If the variance of the future cash flows then does not adjust proportionately to the reduction in asset price, the variance of future cash flows per dollar of asset price will rise, which in turn leads to an increase of volatility in future returns.

They further conclude that, if this process is true and investors correct their expectations, then unanticipated changes in asset prices will be negatively correlated to unanticipated changes in future volatility, i.e., that the impact of u_t^2 on the conditional variance is different for u_t being negative than positive.

To capture these observations, Glosten et al. (1993) introduced a indicator variable I_{t-1} into the GARCH model of the following form:

$$I_{t-1} = \begin{cases} 0 & \text{if } u_{t-1} \geq \mu \\ 1 & \text{if } u_{t-1} < \mu \end{cases} \quad (2.37)$$

Estimating the effect γ of this variable can then be done utilizing the maximum likelihood method.

An integration into the GARCH model changes (2.16) to:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \vartheta I_{t-1} u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (2.38)$$

Or, respectively:

$$\sigma_n^2 = \omega + (\alpha + \vartheta I_{t-1}) u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (2.39)$$

When ϑ is positive, negative shocks have a greater impact on the conditional variance than positive shocks and vice versa. A ϑ equal to zero implies that past positive shocks have the same impact on volatility as past negative shocks (Bauwens et al. 2012). The GJR model consequently allows the volatility persistence to adjust relatively quickly when the sign of the return switches from positive to negative.

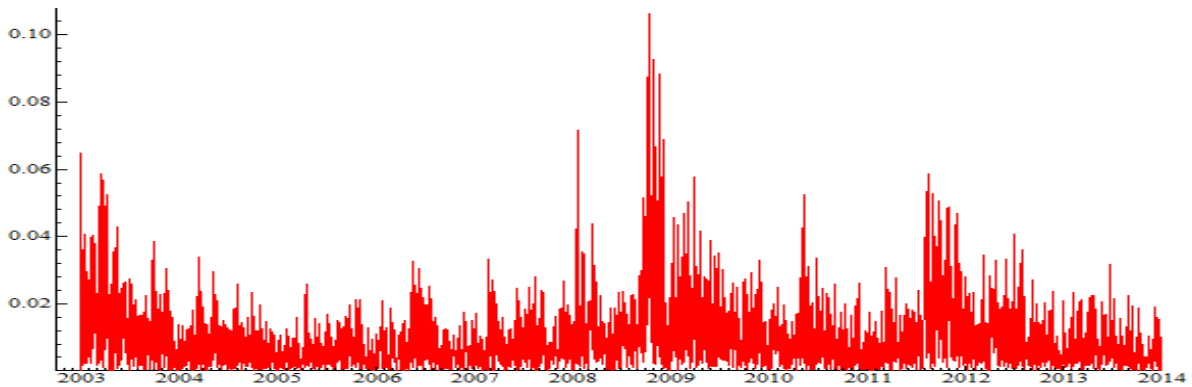
2.1.2.7 APARCH (p,q) - Method

Another model that accounts for the leverage effect is the so-called Asymmetric Power ARCH model (APARCH) developed by Ding et al. (1993), which will be explained in more detail.

Next to the leverage effect, this model also solves for another property of stock returns: the long memory effect, as explained in the IGARCH model.

Other than the GARCH model, Ding et al. (1993) does not consider squared returns u_t^2 , but absolute returns $|u_t|$ of a financial asset, as depicted by Figure 6 for the German CDAX.

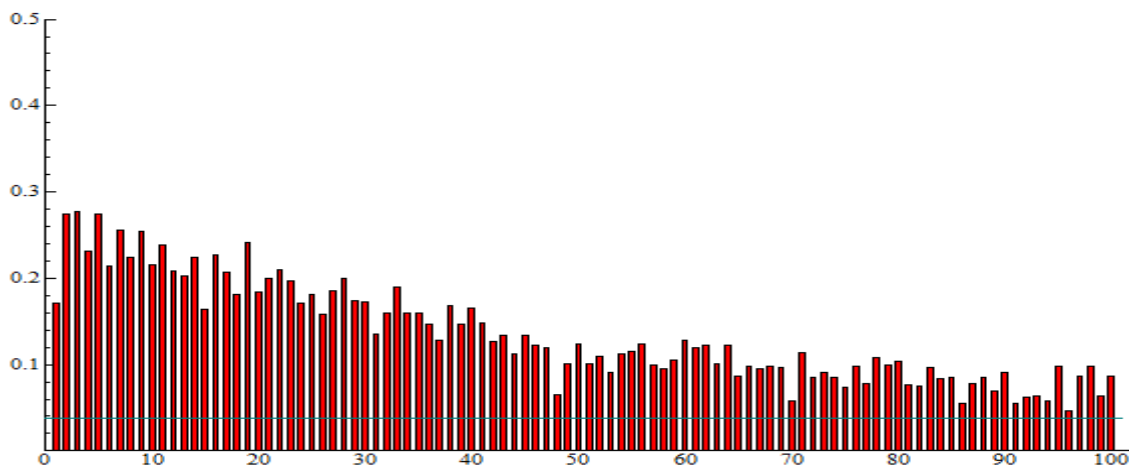
Figure 6: Absolute Returns (CDAX)



Times of high and low volatility can be observed just as in Figure 4, when absolute returns fluctuate to a great extent.

In the context of the long memory property, it then might be useful to apply an autocorrelation analysis as explained by (2.5).

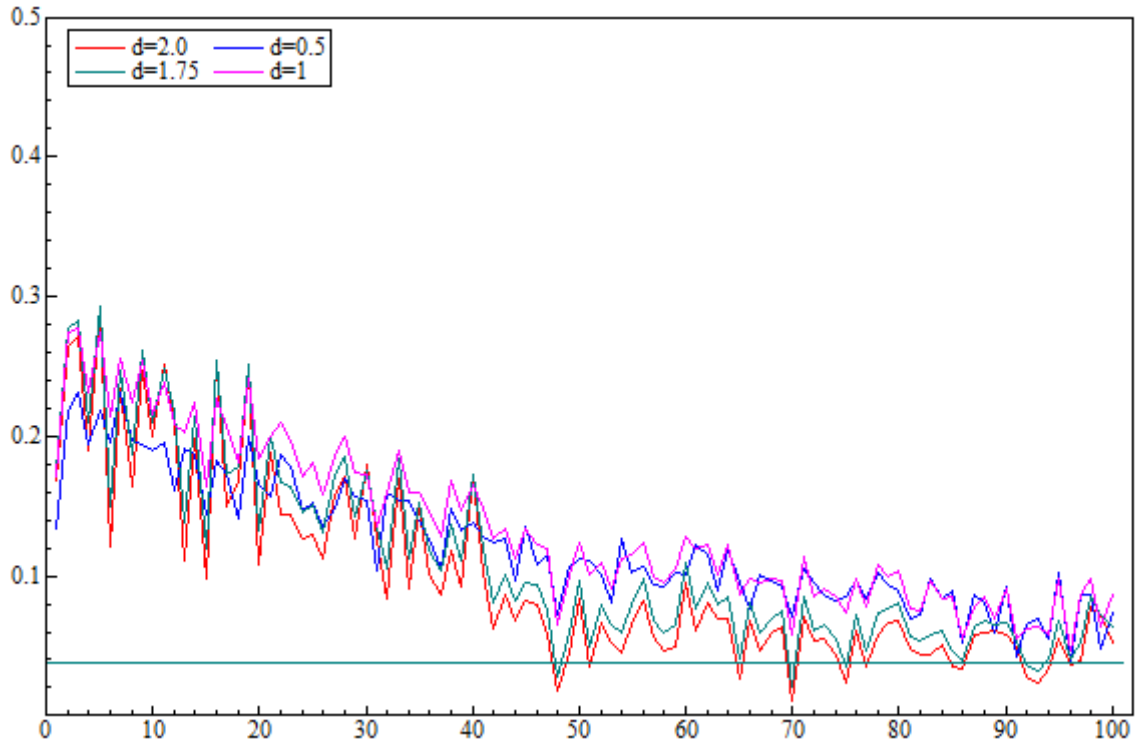
Figure 7: Correlogram of Absolute Returns (CDAX); 2003-2014



With regard to Figure 7, it clearly can be seen that the absolute returns are positively serially correlated for at least 100 lags. In contrast, the squared returns in Figure 5 are not so much serially correlated as absolute. Squared returns are repeatedly insignificantly correlated from lag 45 on.

As a consequence of this observation, Ding et al. (1993) examined these observations for various degrees or powers (d) of absolute returns: $|u_t|^d$. For the values of d equal to 0.5, 1, 1.75, and 2, the following correlogram emerges:

Figure 8: Autocorrelation of Returns to the Power of d



As shown by Figure 8, all the power transformations of the absolute return have significant positive autocorrelations, which supports the hypothesis that market returns have the property of long-term memory. Especially after a lag of 40, absolute returns to the power of 2 and 1.75 have a lower autocorrelation than absolute returns to the power of 1 or 0.5. This might imply that a value of d near one has the strongest autocorrelation. These findings do not, however, concur with the original GARCH model. (2.13) will therefore be amended to incorporate two new features: long-term memory (represented by δ) and leverage effect (represented by φ_n) .

$$\sigma_n^\delta = \omega + \alpha_1(|u_{n-1}| - \varphi_n u_{n-1})^\delta + \beta_1 \sigma_{n-1}^\delta \quad (2.40)$$

with

$$\omega > 0; \delta \geq 0$$

and

$$-1 < \varphi_n < 1$$

An advantage of this equation is that the APARCH model can be easily transformed into other models (Vose 2008). For example, when $\delta = 2$, $\varphi = 0$ and $\beta = 0$, (2.26) equals (2.11) and when $\delta = 2$ and $\varphi = 0$, this equals (2.13). Other models include the TS-GARCH ($\delta = 1$, $\varphi = 0$), GJR-GARCH ($\delta = 2$), T-GARCH ($\delta = 1$), NARCH ($\beta = 1$, $\varphi = 0$) and the Log-ARCH ($\delta \rightarrow 0$)

2.1.2.8 EGARCH (p,q)

Also accommodating the asymmetric effects is the exponential GARCH (EGARCH) model suggested by Nelson (1991). As stated in the review of the GARCH model, Nelson (1991) points out several weaknesses of said model: the restrictive inequality constraints, the evaluation shock persistence, and, finally, the ignoring of asymmetry.

To ensure the non-negativity of the variance, the GARCH model establishes the variance as a linear combination of positive random variables. Additionally, the constraints (2.15a), (2.15b), and (2.15c) exist to assign positive weights to these variables. The EGARCH model has a different approach to this by making the logarithm of the variance $\ln(\sigma_t^2)$ linear in a function of time and lagged returns. The use of this log transformation of the conditional variance ensures that σ_t^2 is always positive. Inequality constraints as in the GARCH model are therefore unnecessary and hence constitute no limitations.

To then account for the asymmetric relationship between stock returns and changes in volatility, Nelson (1991) defines a function $g(u_n)$ incorporating the magnitude and the sign of u_t by making it a linear combination of u_t and $|u_t|$:

$$g(u_t) = \theta u_t + \alpha[|u_n| - E|u_n|] \quad (2.41)$$

In case $0 < u_t < \infty$, $g(u_t)$ is linear in u_t with the slope $\theta + \alpha$. On the other hand, $-\infty < u_t \leq 0$, $g(u_t)$ is linear in u_t with the slope $\theta - \alpha$. This property allows the conditional variance to respond differently to negative shocks than to positive ones.

In this context, it can also be seen that $\alpha[|u_n| - E|u_n|]$ accounts for the magnitude of the change in assets prices and θu_t for the sign of the returns. $E|u_t|$ depends on the assumption made on the unconditional density of u_t .

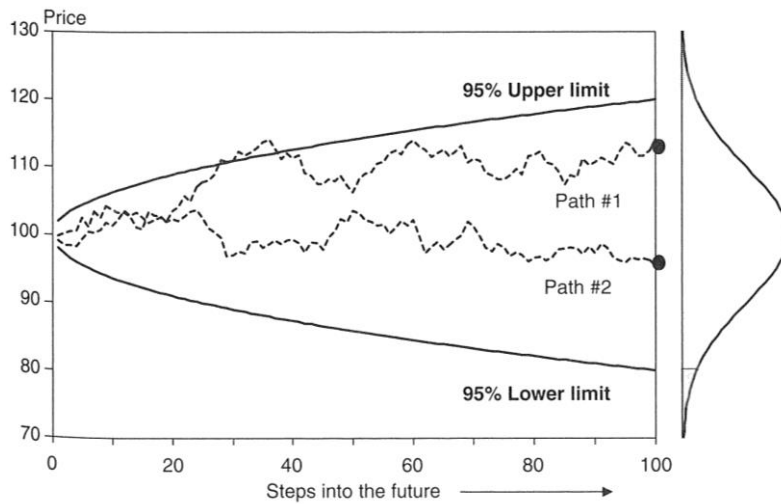
The formula for the conditional variance then is adjusted to:

$$\ln(\sigma_t^2) = \omega + \{\alpha[|u_{n-1}| - E|u_{n-1}|] + \theta u_{n-1}\} + \beta \sigma_{n-1}^2 \quad (2.42)$$

2.1.3 Simulation – Approach

Instead of simply using the historical data to determine value at risk, it is also possible to generate a random price path for the underlying asset. This approach is called the Monte Carlo simulation.

The basis for a Monte Carlo simulation is formed by a random process for the variable of interest. Such a variable is, for instance, the return of a stock index. The mentioned process is then applied repeatedly to simulate a wide range of possible situations. The variables are obtained from a defined probability distribution, which is assumed to be known. It therefore can be said that these simulations reconstruct the entire distribution with an increasing number of simulation runs. (Jorion 2007a, p.308)



To construct a random process that generates the necessary variables, it is essential to specify a particular stochastic model which mimics the behavior of prices. Most models have their origin in the Brownian motion. Other than the models described in the previous section, the Brownian motion proceeds on the assumption that innovations in asset prices, i.e., returns, are uncorrelated over time. Based on this, small changes in asset prices can be derived by the following formula:

$$dS_t = \mu_t S_t dt + \sigma_t S_t dz \quad (2.43)$$

where:

S_t : Asset price

dt : Time increment

μ : Expected value; Mean

The crucial part, which drives the random shocks to the price, is the factor dz . This factor is a random variable that is assumed to be normally distributed and iid.

One possibility to model this is given by:

$$dz = \epsilon \sqrt{\Delta t} \quad (2.44)$$

ϵ : random number; iid.

Additionally, the variance in this model decreases constantly with the time increment dt .

$$V(dz) = dt \quad (2.45)$$

A limitation to this is that jumps in assets prices cannot be modeled accurately. The drift with which the price path grows over time is caused by the mean expression in the formula. The variation in the variance could be simulated with the help of a GARCH process and is assumed to be constant (Jorion 2007a, p.309).

In practice, it is usually more accurate to simulate $\ln S$ rather than S . From Ito's lemma, the process followed $\ln S$ is (Hull 2012, p. 428)

$$d \ln S = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz \quad (2.46)$$

From this, it follows that the changes in the asset's price can be modeled according to:

$$\ln S(t + \Delta t) - \ln S(t) = \left(\mu - \frac{\sigma^2}{2} \right) \Delta t + \sigma \epsilon \sqrt{\Delta t} \quad (2.47)$$

Or equivalently

$$S(t + \Delta t) = S(t) \left(\mu - \frac{\sigma^2}{2} \right) \Delta t + \sigma \epsilon \sqrt{\Delta t} \quad (2.48)$$

2.2 Empirical Studies

2.2.1 Historical Simulation

The historical simulation (HS) is subject to the investigations of Perignon & Smith (2008). They test this approach together with other methods (RiskMetrics, GARCH, FHS, Hybrid HS) for the value at risk with alpha levels of 0.01, 0.025, and 0.05. Their sample consists not, as usual, of equities or indices, but of the daily trading returns of Bank of America, Deutsche Bank, Credit Suisse First Boston, Royale Bank of Canada, and Societe Generale, which they extracted from the respective annual reports. For the observed period of 2001 to 2004, they find that the historical simulation seemingly performs well, but is rejected when taking a closer look at the independence of violations. In general, they find the historical simulation method to display a sluggish behavior regarding volatility shocks, whereas the other tested approaches seem to be more reactive to those.

Similar, Sarma et al. (2003) focus on a selection of value-at-risk approaches, including historical simulation. They test for alpha levels of 0.05 and 0.01 based on observations made between 1990 and 2000 with the underlying indices being the S&P 500 and the S&P CNX Nifty. They apply three tests based on coverage (first order and higher order) and on the loss function and present the "survivors" in their conclusion. At a model selection at 95 percent ($\alpha=0.5$), the historical simulation technique based on a estimation sample of 50, 125, and 1250 days is not rejected, and, in contrast to Perignon & Smith (2008), satisfies the first-order independence property. Testing for higher orders, all methods based on historical simulation are rejected and thus are not selected as appropriate. A similar picture is drawn with a model selection at 99 percent ($\alpha=0.01$).

Another interesting contribution to the literature is made by Angelidis & Skiadopoulos (2008), who test value at risk on the basis of shipping freight rates to determine freight-rate risk. Although at first glance this may not appear to be significantly related to financial markets, these rates are essential for some hedge funds, commodity and energy producers, and traders. Among other models (MA, EWMA, ARCH, EGARCH, APARCH, HS, FHS, EVT) they evaluate the historical simulation method at an alpha level of 0.05 and 0.01 with their

focus on the periods from 1999 to 2004 and 2004 to 2006. The underlying indices represent popular freight markets for dry and wet cargoes (BDI, 4 TC Avg BCI, 4 TC Avg BPI, TD3). Other than Perignon & Smith (2008) and Sarma et al. (2003), they find the “simplest non-parametric method”, i.e. historical simulation, to perform best when determining freight rate risk.

Moreover, historical simulation is examined by Gaglianone et al. (2011), among three other models (GARCH, RiskMetrics, CViaR) with different kind of back-testing procedures. Their sample consists of 1,000 daily returns observed by the S&P 500 from 2003 to 2007. For the HS method with different timeframes, they also note that violations are clustered in time and finally assess the performance as insufficient.

2.2.2 GARCH

A detailed examination of the GARCH model only is conducted by Hung et al. (2008). They set their focus on the performance of the GARCH model under different-distribution assumptions, i.e. normal, student t, and heavy-tailed distribution, while testing for accuracy in energy commodities (WTI, Bent, Heating Oil #2, Propane, NYCGR.) from 1996 to 2006. According to their study, fat tails in return innovation process play an important role and therefore suggest the use of a heavy tail distribution in combination with the GARCH model to come to an accurate value at risk measure. Likewise, in their study from 2010, Füss et al. (2010) compare several value-at-risk approaches with each other and suggest that dynamic approaches (CAViaR, GARCH, RiskMetrics) outperform traditional value-at-risk estimates (Cornish-Fisher-VaR, normal VaR). Using tests of unconditional coverage, independence, and conditional coverage, they evaluate these models based on commodity futures indices from 1991 to 2006. Similar results are found by Gaglianone et al. (2011), who champion the normal GARCH model in their selection. According to Perignon & Smith (2008), however, none of their selected models yield satisfactory results, but the best is the GARCH model with a student t-distribution. Angelidis et al. (2004), Sarma et al. (2003), Wong, W. K. (2009) and others (e.g.: So, Yu (2006), Mokni et al. (2009), Prepic, Unosson (2014)), on the contrary, observe that other GARCH-type models or extensions of these outperform the traditional GARCH model.

Finally, an extremely extensive analysis is conducted by Hansen & Lunde (2005), who propose the question of whether there is anything that beats a GARCH (1,1) model. Based on DM-\$ exchange rate data and IBM return data, they utilize the superior predictive ability (SPA) test and the reality check test to evaluate 330 ARCH-type volatility models. For the sample including the DM-\$ exchange rates, they can answer their question with no, by observing that the GARCH (1,1) model has the best predictive ability. Considering, however, the sample consisting out of the daily returns of IBM, they evaluate the GARCH model as inferior.

2.2.3 RiskMetrics

As mentioned in the review of the historical simulation, Sarma et al. (2003) test several models (EWMA, RiskMetrics, GARCH, Historical Simulation) based on the S&P 500 and the S&P CNX Nifty. Their study reveals that the RiskMetrics approach does not survive all three stages of their elimination process at a significance level of 95 percent, but does in fact

appear to be the best model at a significance level of 99 percent, where it “survives” the first order test, the regression-based test, and is finally selected based on its loss function. For their estimation of value at risk, they use different values for lambda, namely: $\lambda = 0.9$, 0.94, 0.96 and 0.99, where $\lambda = 0.9$ is found to be the best-performing choice.

On the other hand, Giot & Laurent (2003a) find the RiskMetrics approach to perform rather poorly, especially for low-alpha levels such as one, and only acceptable when this level increases to five percent. Moreover, they criticize this approach as too crude, although they note that it does have the advantage of being simple. Furthermore, Giot & Laurent (2003b) find in a later study that value at risk models that are based on the normal distribution, such as the RiskMetrics model, have difficulties in modelling large positive and negative returns, and thus produce inaccuracies.

The model was also part of the comparison of Yu Chuan Huang & Bor-Jing Lin (2004), who examine the forecasting performance of the RiskMetrics, Normal APARCH, and Student APARCH model. Their underlying observations are based on two Taiwanese stock index futures, TAIEX and SGX-DT, from 1998 to 2002. These observations are used to compute value at risk for different levels of alpha ranging from 0.001 to 0.05 and then tested with a different kind of statistical test with a focus on accuracy and efficiency. With regard to the RiskMetrics model, they find that the generated values for value at risk are less accurate than those of the other models and thus reject the RiskMetrics model as well. Similar results were produced by Diamandis et al. (2011), Gaglianone et al. (2011), and Perignon & Smith (2008).

2.2.4 IGARCH

Nikolic-Dorić & Doric (2011) use the RiskMetrics, GARCH, and IGARCH together with the normal and the student t-distribution to compute value at risk. Data is drawn from the Belgrade Stock Exchange index BELEX15 and consists of daily returns from 2005 to 2009. With regard to the IGARCH model, they expect the model to be at least as good as the RiskMetrics approach, as these two models are equal for a mean of zero with $\beta = \lambda$. Since the IGARCH model allows, contrary to the RiskMetrics model, for the estimation of the mean, the IGARCH model is presumed to be less biased. When testing for unconditional coverage, Nikolic-Dorić & Doric (2011), however, find that all models were accepted by the Kupiec's test. Based on these findings, they therefore propose that the IGARCH model cannot outperform the GARCH model. Consequently, they argue that long memory is not crucial for the estimation of the BELEX15's volatility and value at risk.

This interpretation, however, contrasts with that of So & Yu (2006) who argue that the IGARCH model seems to work best for $\alpha = 2.5\%$.

Moreover, the IGARCH model was also in the focus of investigation of Morana (2002). Using daily returns for the DM/US\$ and Yen/US\$ exchange rates, they analyzed the time series from 1986 to 1996 with respect to IGARCH effects. The data generated from this analysis suggests that the IGARCH model is powerful for short-horizon forecasts but becomes less and less useful with an increasing time horizon.

2.2.5 FIGARCH

The next model introduced in the theory part is the so-called FIGARCH model. Research carried out on this model includes Cochran et al. (2012), So & Yu (2006) and (Degiannakis et al. 2013).

Cochran et al. (2012) examine the returns of four metals: copper, gold, platinum, and silver, with the aim of analyzing these returns with regard to long-term dependence. The time period observed for this purpose was from 1999 to 2009, which means that the effects of the financial crisis are captured in these. After testing several long-memory parameters for statistical significance, they derive from this that the FIGARCH model best describes the volatility dynamics of the metals' returns.

Looking at the estimation of value at risk, So & Yu (2006) study seven GARCH models with different kind of properties and apply two different distributions. Taking RiskMetrics as a benchmark, the other models investigated are GARCH, IGARCH, and the FIGARCH model. The basis for this are twelve selected market indices: the All Ordinaries Index (AOI) of Australia, FTSE100, Jakarta Composite (JSX), the Hang Seng Index (HSI), the Kuala Lumpur Composite Price Index (KLSE), KOSPI, NASDAQ, the Nikkei 225 Index (NIKKEI), the Stock Exchange of Thailand Daily Index (SET), the Standard & Poor 500 Index (SP500), the Straits Times Industrial Index (STII) of Singapore, and the Taiwan Stock Exchange Weighted Stock Index (WEIGHT). By doing this, they conclude that the FIGARCH model cannot outperform GARCH models, although autocorrelation plots indicate long memory volatility.

Degiannakis et al. (2013) then also set their focus on the FIGARCH model and its performance in forecasting value at risk. Based on twenty equity markets worldwide with the data covering the period from 1989 to 2009, they test the one-day, ten-day, and twenty-day value at risk for accuracy. Their results suggest that incorporating the FIGARCH model in the estimation process does not improve the accuracy of value at risk.

2.2.6 GJR-GARCH

Current research appears to validate the view that asymmetries play an important role in the dynamics of price movements. Evidence for a negative correlation between stock returns and volatility is borne out by the research of Black (1976) and recently by Bentes et al. (2013b), Ferreira et al. (2007), Tao & Green (2012) and Liu & Hung (2010), which shows that conditional variance is an asymmetric function of the past residuals.

When comparing different GARCH models with each other in the context of their predictive power, Awartani & Corradi (2005) find that asymmetric GARCH models such as the GJR model outperform the models that do not account for asymmetric behavior.

With regard to value at risk, Su, Y. C. et al. (2011) center their discussion on the GARCH model and the model developed by Glosten et al. (1993). To mimic the profits and losses of Taiwanese banks, they simulate two dummy portfolios under different trading assumptions, which consist for simplicity of three asset classes: foreign exchange, equities, and government bonds. These two portfolios are different in size and in their exposures to the three asset classes. This way, they obtain daily profits and losses for a holding period from November 2000 to April 2003, which is equivalent to 617 observations. After estimating value at risk for these portfolios at a 95 percent and 99 percent level, they consequently compare the two different approaches based on the number of violations. The data yielded by this procedure

provides evidence that both models perform very well for the estimation of value at risk. Nevertheless, the GJR model does slightly outperform the GARCH model.

Mokni et al. (2009) also include the GJR model in their evaluation of various value at risk approaches. The issue under scrutiny in their research is the comparison of value at risk derived from selected GARCH (GARCH, GJR-GARCH, and IGARCH) models in times of extreme market conditions (crisis) and normal market conditions (non-crisis), each for the normal, student t, and skewed t-distribution. The dataset used therefore consists of daily returns of the American NASDAQ stock market index, where the timeframe ranges from January 2003 to July 2008. This time series is then split into two sub samples, of which one covers the time during the financial crisis of 2008 and the other the period before the crisis. Based on this data, the performance of the various models is assessed with the aid of the test for sample coverage and Kupiec's LR test. Their key findings suggest that the absolute values of value at risk are obviously higher in the crisis sample than in the non-crisis sample and are both best estimated by the GJR model.

Moreover, Brooks & Persaud (2003) investigate the GJR model in comparison to other value at risk models and find that average proportion of exceedances is lower for the GJR model than for other models, but is outperformed by another asymmetric model.

2.2.7 APARCH

The APARCH model accommodates both the leverage effect and the long memory property. It therefore should perform best in times where these effects are exceptionally distinct in asset returns.

Diamandis et al. (2011) study the APARCH model in great detail and compare it to the well-known RiskMetrics approach. Estimates of value at risk for short and long positions are based on the daily returns of a wide range of stock indices⁴, which cover three main markets. These markets comprise developed countries, Latin America, and Asia/the Pacific. The data covers the period from 1987 to 2009 and therefore also captures the effect of the financial crisis on the volatility dynamics of stock returns. To evaluate the APARCH model in combination with a normal, student t and skewed t-distribution, Diamandis et al. (2011) apply several tests, including Kupiec's test.

The data yielded by this study provides strong evidence that the skewed Student APARCH model considerably improves the estimation of 0.95 and 0.99 as well as 0.05 and 0.01 value at risk for long and short trading positions.

Research by Yu Chuan Huang & Bor-Jing Lin (2004) shows similar results with a focus on the Asian market from 1998 to 2002 and favors the APARCH model compared to the RiskMetrics approach. Moreover, they observe that, at high confidence levels, the value at risk forecasts obtained by applying the student t-distribution are more accurate than those generated using the normal distribution. Note that the skewed t-distribution was not part of their comparison.

⁴ AOI, AC40, DAX30, FTSE20, NIKKEI225, SMSI, FTSE100, S&P 500, Merval, BOVESPA, IPC, SHANGHAI Composite Index, HANG SENG, BSE30, JAKARTA Composite, Kuala Lumpur Composite Index, PSI STRAIT TIMES Industrial Index, KOSPI, CSE Taiwan Weighted Stock Index

However, there is also research which incorporates the APARCH model but does not champion it. As noted earlier, Angelidis & Skiadopoulos (2008), find the historical simulation to perform better than the APARCH model when using it in the context of freight rates.

Another representative of these is Curto & Pinto (2012), who devote their paper to the prediction of volatility during the financial crisis. Considering the daily returns of eight major international stock market indexes: CAC 40, DAX 30, FTSE 100, NIKKEI 225, HANGSENG, NASDAQ 100, DJIA, and S&P 500, they come to the conclusion that the EGARCH model dominates the APARCH.

2.2.8 EGARCH

An extensive comparative study by Angelidis et al. (2004) found that the EGARCH model produces the most suitable value at risk forecasts for most of their observed asset markets. They come to this conclusion by comparing the GARCH, TARARCH, and EGARCH model based on different-distribution assumptions (Normal, -student t and GED-distribution) and different alpha levels ($\alpha=0.05$ and $\alpha=0.01$). The markets they focus on are represented by their major indices, which are the S&P 500, FTSE 100, NIKKEI 225, DAX 30, and the CAC 40. The time period observed ranges from 1987 to 2002. Building up an evaluation framework based on a quantile loss function, Angelidis et al. (2004) then find that an EGARCH model in combination with a student t-distribution leads to the most adequate value at risk measures. When changing the focus from equities to freight rates, Angelidis & Skiadopoulos (2008), however, find that the model is mostly outperformed by simpler approaches.

Next to these findings, Brooks & Persaud (2003) investigate the effect of asymmetries in stock index returns on value at risk. The returns were sampled from the Hang Seng Price Index, Nikkei 225, the Singapore Straits Times Price Index, the South Korea SE Composite Price Index, the Bangkok Book Club Price Index, and the S&P 500 and concentrated on the period from 1985 to 1999. The evaluation was then based on the back-testing framework proposed by the BASEL Committee. The essential observation made is that the average proportion of violations is lower for the EGARCH model than for the other selected models, but are in total outperformed by a non GARCH model, called the semi-variance model.

2.2.9 Monte Carlo Simulation

The Monte Carlo simulation is part of a generic approach to value at risk. By simulating return data based on past information or estimations, the same concept of the historical simulation can be applied and a value at risk derived.

Comparing the most eminent representatives of each approach category, namely historical simulation, variance-covariance approach⁵ based on normal distribution, and the Monte Carlo Simulation, Dedu & Fulga (2011) find that the best method for value at risk estimation and forecasting is the basic Monte Carlo method. In their paper, particular emphasis is given to the portfolio optimization problem based on a mean-value at risk framework instead of a mean-variance approach. For this reason, they compute value at risk based on the three different methods with a time horizon of 30 days and an alpha level of five percent. Their underlying

⁵ No GARCH model is applied. Variance estimation is done according to (2.2)

portfolio consists of 10 assets from Bucharest Stock Exchange: ALBZ, ATB, BIO, BRK, IPRU, OLT, SIF5, SNP, TBM, and TLV.

Rejeb et al. (2012) similarly investigated the same three approaches and added a fourth approach, which represents an adjustment to the historical simulation, called the bootstrapping method. Their data covers the time between 1999 and 2007 and consists of daily data concerning the US Dollar, the Euro, and the Japanese Yen returns in relation to the Tunisian dinar. Based on their initial analysis, they find that violations for all approaches lay very close together, but then can conclude that traditional Variance-Covariance is the most appropriate method, even though the Monte Carlo simulation provides a more robust estimation of value at risk.

Po-Cheng Wu et al. (2012) conduct research based on an equally weighted portfolio of Equity (TAIEX Index), Bonds (ten-year Taiwanese government bond), Foreign Exchange (NTD/USD), and Options (TAIEX Index option, put) to assess the performance of the GARCH model, RiskMetrics approach, historical simulation, and Monte Carlo Simulation. Their data is drawn from weekly returns from 2002 to 2009 and thus consists of 417 observations. Based on this data, they find that the historical simulation performs best and report that the Monte Carlo approach is the second-best model to forecast value at risk.

Nonetheless, Fabozzi et al. (2013) pointed out that it is important to be aware of the model risk that is inherent to the Monte Carlo simulation. They see various sources for this kind of risk, including incorrect specifications of the model that generates the returns, estimated parameters such as variance deviating from their true value, or the assumption of an unrealistic underlying distribution.

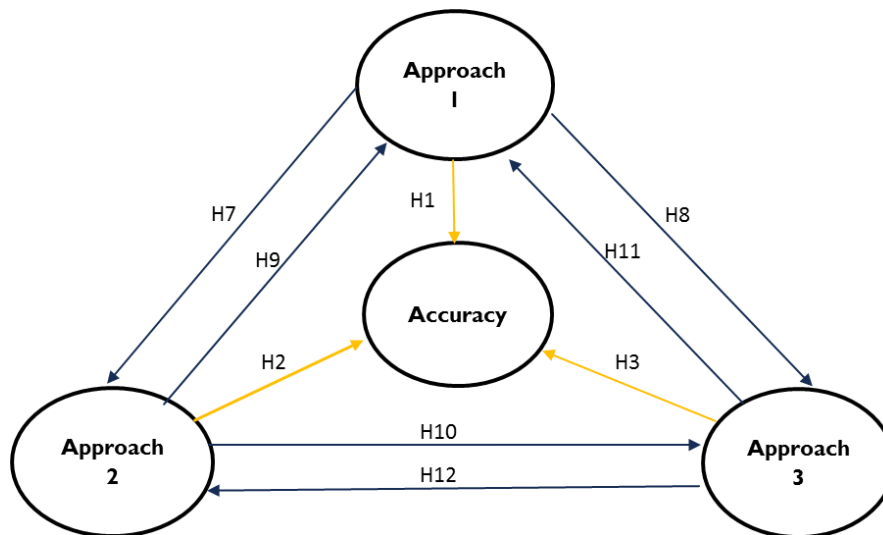
This statement is, however, not only valid for the Monte Carlo simulation but for all models analyzed in this paper.

3. Research Methodology and Methods

3.1 Research Hypotheses

As shown in in the previous section, there are three categories of value at risk approach. To offer a valuable comparison, there will be at least one approach analyzed per category, where emphasis will be put on the parametric approaches, which are mostly championed by the literature.

Since the overall aim of the thesis to identify and rank the most accurate approaches to determine value at risk in normal and extreme conditions, the individual hypotheses are structured as follows:



where:

H1: Approach 1 predicts with sufficient accuracy the maximum loss of a stock portfolio over a period of time for a given probability in normal market conditions

H2: Approach 2 predicts with sufficient accuracy the maximum loss of a stock portfolio over a period of time for a given probability in normal market conditions

HX: Approach X predicts with sufficient accuracy the maximum loss of a stock portfolio over a period of time for a given probability in normal market conditions

and:

H7: Approach 1 predicts the maximum loss of a stock portfolio over a period of time for a given probability in normal market conditions better than Approach 2

HY: Approach X predicts the maximum loss of a stock portfolio over a period of time for a given probability in normal market conditions better than Approach Y.

The same will then be applied for extreme market conditions. That way, the result will be ranking of VaR approaches in extreme and normal market conditions.

The literature reviewed suggests the following approaches for comparison:

Non-Parametric	Parametric	Semi-Parametric
<ul style="list-style-type: none"> Historical Method 	<ul style="list-style-type: none"> ARCH GARCH RiskMetrics IGARCH FIGARCH APARCH EGARCH GJR 	<ul style="list-style-type: none"> Monte Carlo Simulation

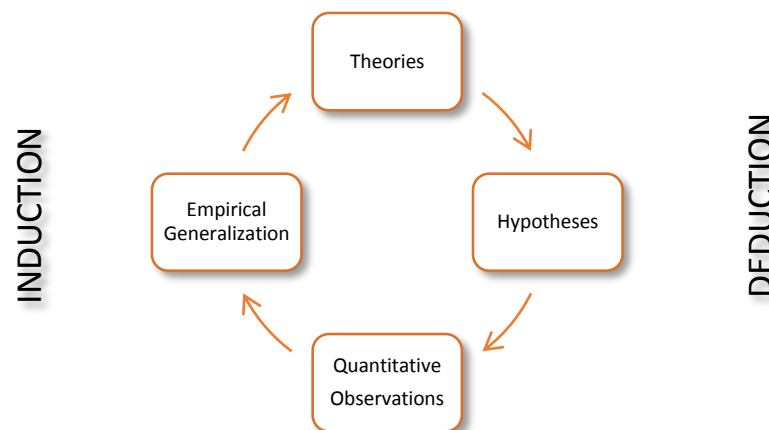
The number of hypotheses will therefore increase with the number of compared approaches.

3.2 Research Philosophy

The aim of this research is to test a theory to develop hypotheses, which will then provide material for the development of laws: i.e., which VaR approach is suitable in which situation. Since the theories that will be tested are based on mathematical and statistical models, the tests themselves will be purely quantitative on the basis of facts, i.e. past share price movements. This way, it will be ensured that values or beliefs will not distort the outcomes of the research. The research will therefore be objective rather than subjective. Moreover, the analysis method will be structured and diversified, since there will be at least two tests conducted per VaR approach.

Given these major characteristics, the research philosophy applied can be classified as positivism (Bryman 2008).

Other attributes of positivism also fit the research method chosen. For instance, an inductive approach will be applied as well as a deductive one, as illustrated by Walter Wallace's *Wheel of Science*:



From the review of relevant theories about value at risk, hypotheses will be constructed about which approach to calculate VaR is most accurate. These hypotheses will then be tested by quantitative observation. After that, generalizations can be made about which approach is most suitable. These generalizations can then be compared to existing theories and an own theory can be suggested.

3.3 Research Strategy

The research strategy is to conduct a quantitative study which is both descriptive and comparative. Furthermore, accuracy in normal and extreme market conditions will be described independently.

As mentioned earlier, the study will also be comparative, since the overall aim is to create a ranking of the various value at risk approaches with regard to their accuracy in normal and extreme market conditions.

To do this, the first step will be to collect the share price data of chosen stock indices (see 3.6 Data Collection, Editing, Coding and Analysis) and to convert them into daily returns. From these returns, other variables such as variance, mean, and kurtosis for a specified time period

will be derived, incorporating different estimation approaches. The derived variables will then be used to calculate value at risk in the various approaches.

The next step is the comparison of the forecasted maximum loss by VaR with the actual loss respectively return, a process called back testing. To achieve valuable results, the following test will be applied to judge the accuracy of value at risk:

Kupiec's proportion of failures test

One of the most prevalent techniques to assess the accuracy of a value at risk model is the so-called proportion of failures test developed by Kupiec. The basis for this test is provided by the analysis of how frequently asset losses exceed the estimated value at risk in form of a statistical hypothesis test. The null hypothesis H_0 which is to be tested is hence that the frequency of empirical exceptions is consistent with the expected theoretical exceptions specified by the value at risk alpha level.

$$H_0: \alpha = \hat{\alpha} \quad (3.1)$$

where :

$$\hat{\alpha} = \frac{x}{N}; \text{ failure rate}$$

The alternative hypothesis H_1 is that the frequency of empirical exceptions is higher than the alpha level.

$$H_1: \alpha > \hat{\alpha} \quad (3.2)$$

Using these hypotheses, a preliminary analysis can be conducted by determining the absolute value of:

$$|\alpha - \hat{\alpha}| \quad (3.3)$$

which depicts the deviation of the empirical failure rate from the expected failure rate. The smaller this number, the more accurate the model, and vice versa. This way, a ranking can be conducted where the lowest value is represented by the best rank (Rank 1) and the highest values by the worst rank (Rank 21). However, to see whether the deviation is significant, further test statistics have to be computed.

Due to the fact that this test does not rely on any other hypotheses, it is also known as the unconditional coverage test. In case the null hypothesis is true, the number of violations and then the number of exceptions follow a binomial distribution. The probability that x violations are observed in a sample of N is given by(Jorion 2007b):

$$prob(x|\alpha, N) = \binom{N}{x} \alpha^x (1 - \alpha)^{N-x} \quad (3.4)$$

x: number of violations

N: sample size

For example, the probability of 16 exceptions violating a value at risk model with the alpha level of 5 percent in a sample of 500 daily returns can be determined this way:

$$prob(11|0.05,250) = \binom{500}{16} 0.05^{16} (1 - 0.05)^{500-16} = 0.0144 \approx 1.44\%$$

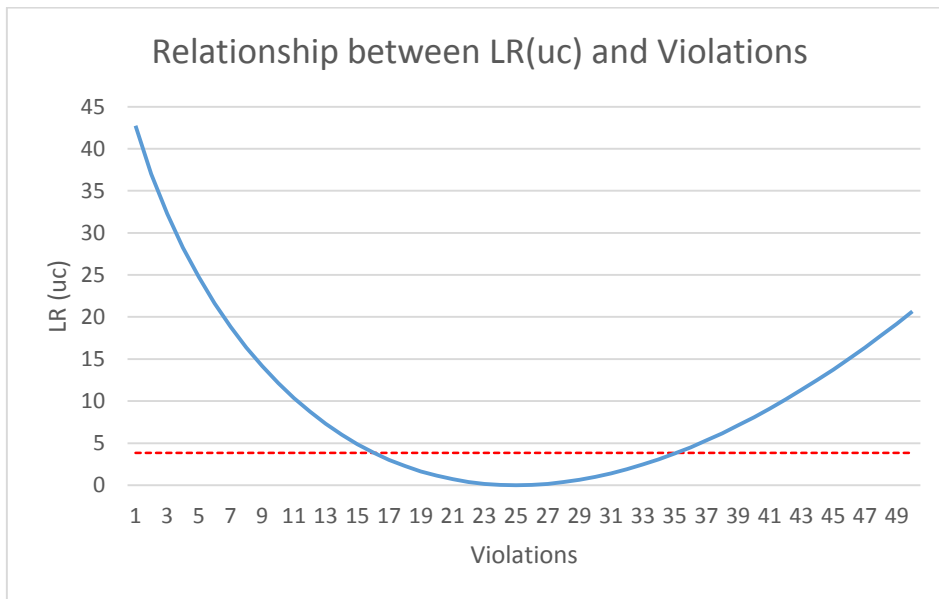
To clarify whether the failure rate π is significantly different from alpha, the cumulative binomial distribution can be utilized to calculate an interval within the number of violations must fall for the test to accept the null hypothesis. For this purpose, Kupiec suggested the use of the tail points the log-likelihood-ratio. The test statistic LR_{uc} fits the null hypothesis and the alternative hypothesis to the data given and then computes the difference times two (Jorion 2007b):

$$LR_{uc} = -2 \ln \left[\frac{\alpha^x (1 - \alpha)^{N-x}}{\hat{\alpha}^x (1 - \hat{\alpha})^{N-x}} \right] \quad (3.5)$$

This test statistic (LR_{uc}) is then asymptotically chi-square distributed with one degree of freedom. For a significance level of 95%, the null hypothesis would be rejected when $LR_{uc} > 3.841$.

For a sample size of 500 daily returns, the following relationship between LR_{uc} and the number of violations can be observed:

Figure 9: LR(uc) and Violations



With the aid of Figure 9, it is possible to see that the model will be rejected when the number of violations is greater than 35 or smaller than 17, which means that values of x greater than or equal to 36 indicate that the value at risk model understates the probability of large losses and values of x smaller or equal to 16 overstate value at risk. This way, non-rejection intervals for various specifications of alpha and sample sizes can be determined. Table 1 gives some examples for an alpha level of 5% and selected sample sizes.

Table 1: Non-rejection Intervals for Number of Violations x

Sample Size	Non-rejection Interval for Number of Violations x
250	$6 < x < 20$
500	$16 < x < 36$
1000	$37 < x < 65$
1250	$47 < x < 79$
1500	$59 < x < 93$
2750	$115 < x < 161$

Nevertheless, it as to be mentioned that these kinds of tests are not completely free of errors. In fact, this sort of test is subject to two types of errors:

	H_0 is correct	H_0 is wrong
Accept H_0	OK	“Type 2 error” β -error
Reject H_0	“Type 1 error” α -error	OK

The α -error represents the risk of rejecting a value at risk model that is correct and the β -error corresponds to the risk of not rejecting an incorrect model. The α -error depends on the number of violations that are used to accept the model and is mostly predetermined. In this case, it was set to 5% since the test is conducted for a significance level of 95%. The probability of rejecting a value at risk model that is correct was hence set to 5%.

Christoffersen Test

Although a powerful instrument, the Kupiec’s test has some limitations. Most important is its sole emphasis on the number of violations without taking into account the order in which they occur.

It does not, therefore, consider if exceptions are clustered around a special point of time or not, i.e., if the violations are independent of each other. In this case, given that a violation occurred on day $t - 1$, the conditional probability of having another exception on day t will be higher than average. Using the Kupiec’s test in this context might thus lead to the acceptance of a model alternating periods in which value at risk is underestimated with periods in which it is overestimated (Sironi & Resti 2007).

On the other hand, if the value at risk model can react promptly to new information, the probability of a violation at time t should be independent of any observed violations observed on day $t - 1$.

To also test for the phenomena described, a conditional coverage test can be applied.

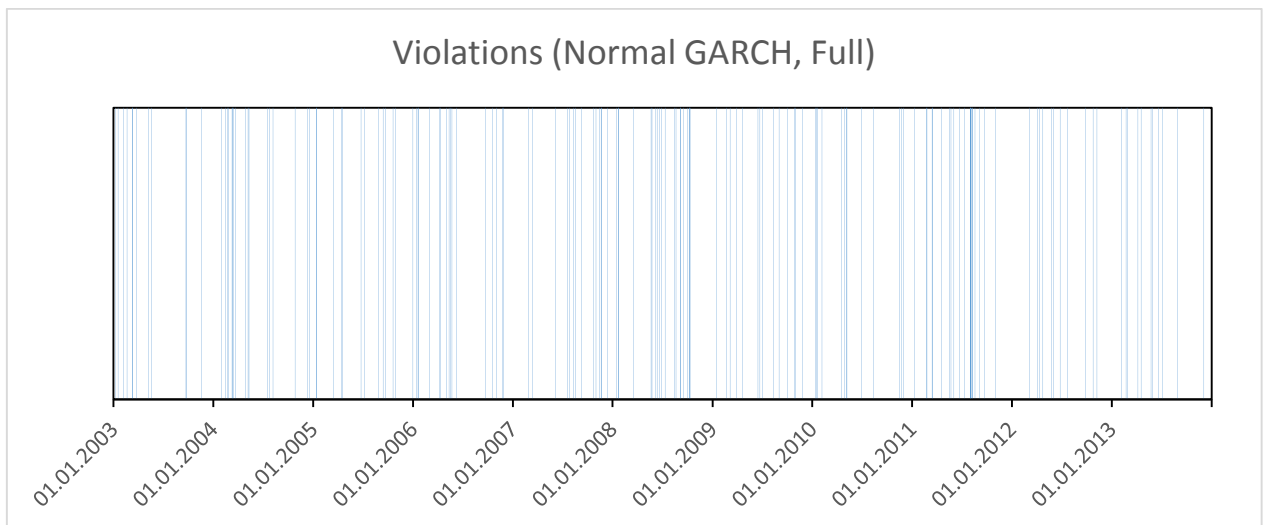
A well-known test for conditional coverage has been developed by Christoffersen. The test builds on the LR_{uc} test statistics in such a way that it also incorporates serial independence. It thus tests whether the probability of an exception on any day depends on the previous day's violation or non-violation (Sironi & Resti 2007).

For this reason, an indicator variable can be defined as follows:

$$I_t = \begin{cases} 1 & \text{violation was observed} \\ 0 & \text{no violation was observed} \end{cases} \quad (3.6)$$

An example of this can be found in Figure 10:

Figure 10: Violation Clustering



It can be seen that some violations seem to appear individually and some cluster around a certain point of time. This is especially notable in this example because the second half of 2008 seems to be clustering and violations might not be independent of each other. Even if this value at risk model passes the unconditional coverage test, it still is possible to reject it when testing for independence.

Using the indicator function, a matrix can be created that maps all kinds of states that can occur:

Table 2: Conditional Exceptions

	$I_{t-1} = 0$	$I_{t-1} = 1$
$I_t = 0$	n_{00}	n_{10}
$I_t = 1$	n_{01}	n_{11}

Where:

n_{00} is the number of observations where there was no violation on day t and on day $t - 1$

n_{10} is the number of observations where there was a violation on day t but not on day $t - 1$

n_{01} is the number of observations where there was no violation on day t but on day $t - 1$

n_{11} is the number of observations where there was a violation both on day t and on day $t - 1$

From these possible states, one can derive the probability that a violation of value at risk takes places at time t without this being the case on the previous day $t-1$, as follows:

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}} \quad (3.7)$$

Or the probability that a violation of value at risk takes places at time t , this being also the case on the previous day $t-1$:

$$\pi_1 = \frac{n_{11}}{n_{10} + n_{11}} \quad (3.8)$$

and respectively:

$$\pi = \frac{n_{11} + n_{01}}{n_{10} + n_{11} + n_{00} + n_{01}} \quad (3.9)$$

When the exceptions are independent, the probabilities of π_0 and π_1 should be equal. Similar to (3.3), a preliminary analysis can be done by looking at the absolute deviations:

$$|\pi_0 - \pi_1| \quad (3.10)$$

Here, the rule also applies that small values indicate independence of violations and bigger ones dependence.

To test the hypothesis that the exceptions are significantly independent, Christoffersen also suggest the use of a log-likelihood-ratio.

The test statistics LR_{ind} is then :

$$LR_{ind} = -2\ln \left[\frac{(1 - \pi)^{n_{00}+n_{10}} \pi^{n_{01}+n_{11}}}{(1 - \pi_0)^{n_{00}} \pi_0^{n_{01}} (1 - \pi_1)^{n_{10}} \pi_1^{n_{11}}} \right] \quad (3.11)$$

This test statistic (LR_{ind}) is then just as (LR_{ind}) asymptotically chi-square distributed with one degree of freedom. A decision rule can thus be defined in accordance with the quantiles of the chi-square distribution.

It is, however, important that this test has some limitations as well. The test itself is based on a first-order Markov chain, which means that the test is only able to detect clustering in violations if exceptions are actually consecutive. In cases where there are small gaps in between two consecutive violations, the Christoffersen's test might fail. To detect this kind of clustering, the formula has to be extended to a higher-order Markov chain (Alexander 2009).

Joint Test

To analyze both properties of a well-defined value at risk model, it is also possible to combine these two tests into a joint test. This test then accounts for the number of significant violations as well as their independence. It is, therefore, a test for conditional coverage. The way to do this is simply to add up the two separate test statistics (LR_{ind}) and (LR_{ind}):

$$LR_{cc} = LR_{uc} + LR_{ind} \quad (3.12)$$

The sum of (LR_{ind}) and (LR_{ind}) then form a new test statistic LR_{cc} , which is asymptotically chi-square distributed but with two degrees of freedom instead of one. The cutoff value at a significance level of 95 percent for this is then 5.991. Any value bigger than 5.991 leads to the rejection of the tested model. It is, however, important to note that there is the possibility of a model passing the joint test while being rejected by the Kupiec's test or Christoffersen's test. All three tests must therefore be considered to come to a valid conclusion.

Since this study will also be longitudinal, the three described tests will be applied to the various samples described in the next section.

3.4 Ethical Issues and Procedure

Research Ethics

As previously explained, this research will use a longitudinal quantitative design. The data used to conduct the research is publicly available and free to use. Permission for the utilization of this data is therefore not required. Also, the computation methods for value at risk can be easily found in literature, as well as the approaches to test its accuracy. The research can therefore be repeated by an independent researcher. An issue might be the choice of the sample indices. To avoid any bias, indices of with a large number of companies from all business sectors will be chosen.

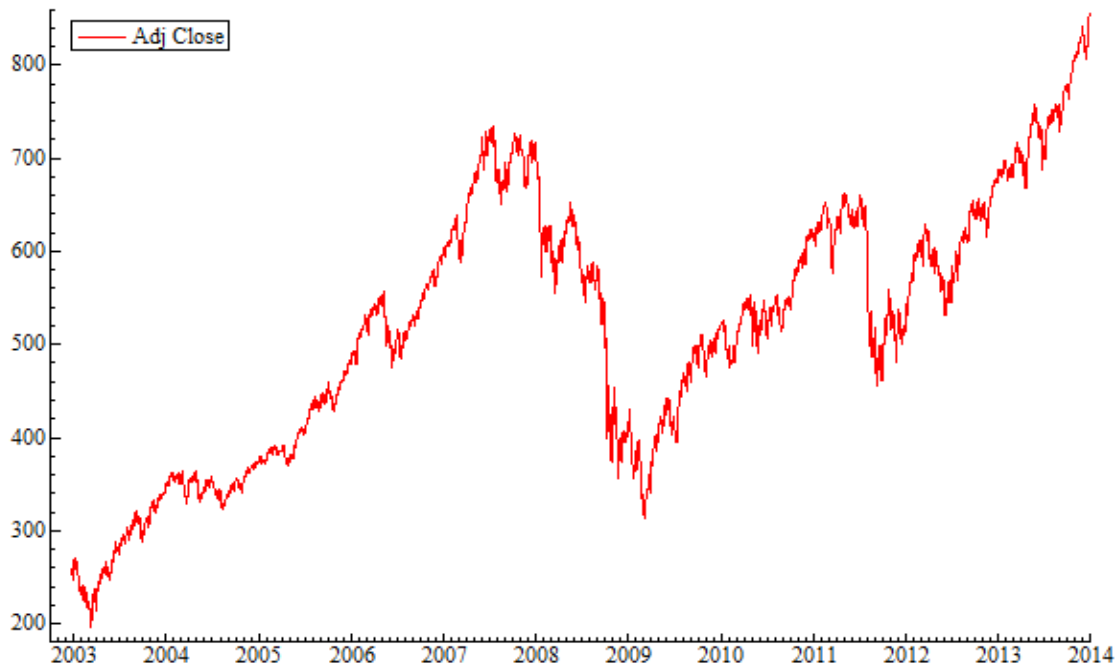
3.5 Population and Sample

For the research conducted, the overall population is defined as all companies that are listed on the German stock exchange in Frankfurt am Main, currently 492 companies. Compared to

the biggest stock exchange in London, currently with 2455 listed companies, this is only about one-fifth.

The sample used for this study will, however, be equal to the population, since the Composite DAX index will be used to test the VaR approaches. This index consists of all companies listed at the German stock index. This would therefore represent a quota of 100% in a non-probability sampling framework.

Figure 11: Price Chart



The accuracy of the estimations and test conducted in this paper is, however, essential for the number of daily returns observed. The data for the study considers daily adjusted closing prices of CDAX Index from 1st January 2003 to 31st December 2013 to derive the daily returns as described in formula (1.4). This equals 2826 observed returns in total.

Next to this overall sample, two sub-samples of different time periods will be considered to enable a reflective comparison of the various value at risk approaches within times of normal and extreme market conditions.

For this reason, the considered time period will be divided into the period from 1st January 2003 to 31st December 2007 and the period from 1st January 2008 to 31st December 2013, where the first one equals approximately 1260 observations and the later one approximately 1512 observations. The sample from 2008 to 2013 is particularly interesting due to the fact that the global financial crisis occurred within this time period.

For all price series, daily returns are defined as described in formula (1.4). Graphical analysis (price chart, daily returns, density, histogram of the returns compared to a normal distribution, and QQ-plots against the normal distribution) for the index are given in Figure 11, Figure 12 and Figure 13.

Notably, Figure 13 shows that the returns are not fully normal-distributed and exhibit long tails. Taking a closer look at the QQ-plot even indicates that these fat tails are not symmetric.

Moreover, Figure 12 shows that the times series exhibit volatility clusters: periods of low volatility are followed by periods of high volatility.

Figure 12: Return Series

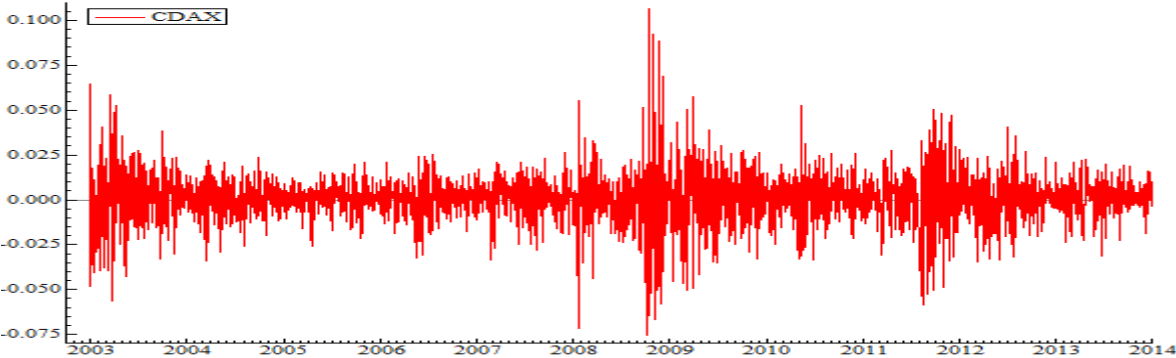
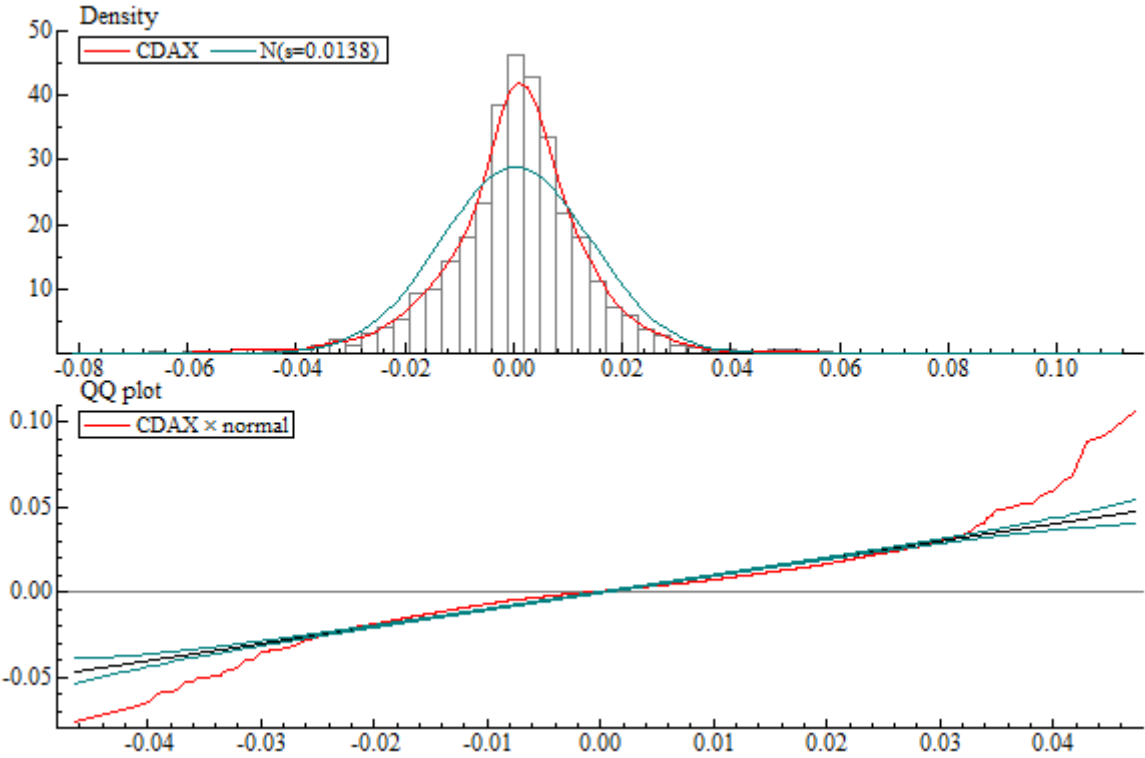


Figure 13: Histogram, Density Fit and QQ-Plot



To assess these observations, descriptive statistics are reported in Table 3:

Table 3: Descriptive Statistics

Observations	2826
Minimum	-0.075522
Mean	0.00042241
Maximum	0.1064
Std. dev.	0.013841
Skewness	-0.058609
Excess Kurtosis	5.5412
Jarque-Bera	3617.1
Q(5)	30.0575
Q(10)	49.2935
Q(20)	80.0063

As expected, the mean of the overall sample is close to zero, whereas the standard deviation was observed to be rather high, which might suggest a higher level of fluctuations regarding the daily returns of the CDAX for the considered time period. Since the skewness is unequal to zero, it can be assumed that the underlying distribution is not completely symmetric. In fact, the skewness is slightly negative and thus suggests that the distribution is to some extent skewed to the left. This means that the tail on the left side of the distribution might be longer than the right one. In addition to that, a kurtosis of 8.54 can be observed, which is an excess of 5.54 compared to the normal distribution. As this number is positive, this indicates a more cuspidal or leptokurtic density function, as depicted by Figure 13. These findings are supported by the Jarque-Bera normality test (Jarque & Bera 1987), which clearly rejects the null hypothesis of a normal distribution for a significance level of 5%⁶. Table 3, furthermore, shows the Q statistics of Ljung-Box portmanteau tests⁷ for the lags five, ten, and twenty of the squared returns. This test examines whether the autocorrelation of the given time series is significant or not. Under the null hypothesis of no serial correlation, Q(l) is distributed asymptotically as a chi-squared distribution with the degrees of freedom being equal to the number of lags. It can be seen that all Q statistics indicate serial correlation for the squared returns at significance levels of five and one percent. In short, the graphical analysis and descriptive statistics suggest that the distribution is best described by leptokurtic distribution with long tails and volatility clustering.

⁶
$$JB = N \left(\frac{(Skewness)^2}{6} + \frac{(Excess\ Kurtosis)^2}{24} \right) \sim \chi^2_2$$

A different picture is presented when looking at the two sub-samples individually.

Table 4: Descriptive Statistics Sub-Samples

	2008 - 2013	2003 - 2007
Observations	1531	1292
Minimum	-0.075522	-0.056639
Mean	0.00011387	0.00081298
Maximum	0.1064	0.06828
Std. dev.	0.015665	0.011219
Skewness	0.0045743	-0.10816
Excess Kurtosis	5.1302	3.3676
Jarque-Bera	1679	613.02
Q(5)	22.4592	31.9819
Q(10)	33.37	36.6127
Q(20)	54.9075	43.1967

In comparison, the two observed periods have some essential differences in their descriptive statistics. For obvious reasons, the range between minimum and maximum in the two samples is quite different, since one sample incorporates the financial crisis of 2008. Consequently, their standard deviation is distinctive. As noted, the overall sample is a standard deviation of 0.013841 and is considered as high but is also observed to be even higher in the crisis sample and respectively lower in the pre-crisis sample. It is interesting that excess kurtosis increased more than 50 percent in the period of 2008 to 2013 compared to the period of 2003 to 2007. Even more significant is the change in skewness. Being negative in the overall sample (-0.058609), the skewness changed sign in the crisis period and amounts to a small positive value (0.0045743), which indicates a distribution skewed slightly to the right, whereas in the pre-crisis period the distribution was skewed to the left with a value of (-0.10816). Of course, these differences then have an impact on the Jarque-Bera test statistics, which still indicate non-normality. The Q statistics of the squared returns also still suggest autocorrelation in both periods.

3.6 Data Collection, Editing, Coding and Analysis

The quantitative study is based on historic daily share price observations of the German CDAX. This data will be retrieved from the Yahoo-Finance database. This data will provide the daily closing prices of the CDAX adjusted for dividends, splitting, or similar events to include all kinds of returns.

To come from this raw data to value at risk, the strategy of section 3.3 will be applied incorporating the approaches introduced in the literature review. This will be done utilizing Microsoft's Excel, which to a certain degree allows for the calculation of value at risk as well as testing for its accuracy.

Next to Microsoft Excel, the OxMetrics 7 software suite will be used to derive value at risk from the daily returns. OxMetrics is a menu-based system with a user-friendly graphical user interface, which is specialized towards econometrics, statistics, and financial analysis. The OxMetrics program itself, however, is just the front-end platform with which several software modules can be integrated. The module used to conduct this research is the G@RACH 7 module, which was developed by major researchers in the field of volatility modeling. The module hence has its focus on the estimation and forecasting of univariate and multivariate ARCH-type models.

4. Data Analysis

This part of the study provides details of the computed research data that was conducted using the described methods in the methodology part. The test statistics LR_{uc} , LR_{ind} and LR_{cc} of the selected back test are reported as well as the preliminary analysis of the absolute differences explained by (3.3) and (3.10) in section 3. The structure of this section will orient itself to the timeframes given by the tested samples, i.e. pre-crisis, crisis, and full sample.

4.1 Analysis of the period from 2003 to 2013

Based on the absolute differences of the failure rate to the target rate (=VaR alpha level) as well as the absolute difference of the proportion of consecutive and non-consecutive violations, two individual rankings can be conducted and an average rank derived. The highest rank, i.e. smallest absolute difference, is numbered with one and the lowest rank numbered 24.

Table 5: Ranking (2003-2013)

Ranking Based on Absolute Differences (2003-2013)			
	Failure Rate Ranking	Independence Ranking	Avg. Rank
HS (100)	9	21	15
HS (250)	4	22	13
HS (500)	3	24	13.5
GARCH (Normal Distribution)	8	3	5.5
GARCH (Student t-distribution)	21	1	11
GARCH (Skewed t-distribution)	11	8	9.5
RiskMetrics	18	2	10
IGARCH (Normal Distribution)	11	8	9.5
IGARCH (Student t-distribution)	22	13	17.5
IGARCH (Skewed t-distribution)	6	6	6
FIGARCH (Normal Distribution)	13	5	9
FIGARCH (Student t-distribution)	23	11	17
FIGARCH (Skewed t-distribution)	10	12	11
EGARCH (Normal Distribution)	1	4	2.5
EGARCH (Student t-distribution)	16	7	11.5
EGARCH (Skewed t-distribution)	15	10	12.5
GJR (Normal Distribution)	14	16	15
GJR (Student t-distribution)	19	18	18.5
GJR (Skewed t-distribution)	6	17	11.5
Monte Carlo (Normal Distribution)	24	20	22
Monte Carlo (Student t-distribution)	2	23	12.5
APARCH (Normal Distribution)	17	15	16
APARCH (Student t-distribution)	19	18	18.5
APARCH (Skewed t-distribution)	5	14	9.5

Table 5 depicts these ranking and reveals that the exponential GARCH model in combination with a normal distribution, the Monte Carlo simulation based on a student t-distribution, and the historical simulation with a rolling estimation window of 500 days have the three lowest

deviations from the VaR alpha when compared to the actual failure rate. When looking at the independence of violations, the GARCH model in combination with a student t-distribution and a normal distribution as well as the RiskMetrics approach yield the smallest absolute difference between π_0 and π_1 .

Given that for each category the top three are distinct, it therefore not surprising that the average ranking of these do not reflect the overall top three (Normal-GARCH, Normal-EGARCH, and Skewed-IGARCH).

This, however, is also a sign of the greatest limitation of creating a ranking this way due to the fact that, with this procedure, it might be possible that an approach ranked very high in one ranking and very low in another ranking gets good results in the average rating.

To see whether the deviations are significant, the test statistics of the Kupiec's, Christofersen's, and the Joint test have to be analyzed. Table 6 depicts these values for the selected approaches.

Table 6: Test Statistics (2003-2013)

Test Statistics for the Selected VaR-Approaches (2003-2013)							
<i>Methods</i>	<i>Failure</i>		π_0	π_1	π	LR_{ind}	LR_{cc}
	<i>Rate</i>	LR_{uc}					
HS (100)	0.0425	3.5068	0.0392	0.1167	0.0425	11.9107	15.4176
HS (250)	0.0443	2.0197	0.0400	0.1360	0.0443	17.5392	19.5589
HS (500)	0.0450	1.5430	0.0397	0.1575	0.0450	25.1177	26.6606
GARCH (Normal Distribution)	0.0574	3.1010	0.0579	0.0494	0.0574	0.2125	3.3135
GARCH (Student t-distribution)	0.0648	11.9931	0.0652	0.0601	0.0648	0.0733	12.0665
GARCH (Skewed t-distribution)	0.0588	4.3696	0.0598	0.0422	0.0588	0.9674	5.3371
RiskMetrics	0.0641	10.9150	0.0636	0.0718	0.0641	0.1852	11.1002
IGARCH (Normal Distribution)	0.0588	4.3696	0.0598	0.0422	0.0588	0.9674	5.3371
IGARCH (Student t-distribution)	0.0652	12.5498	0.0667	0.0435	0.0652	1.6983	14.2480
IGARCH (Skewed t-distribution)	0.0567	2.5450	0.0575	0.0438	0.0567	0.5705	3.1155
FIGARCH (Normal Distribution)	0.0595	5.0810	0.0588	0.0714	0.0595	0.4291	5.5102
FIGARCH (Student t-distribution)	0.0680	17.4160	0.0665	0.0885	0.0680	1.2666	18.6826
FIGARCH (Skewed t-distribution)	0.0581	3.7094	0.0568	0.0793	0.0581	1.2971	5.0065
EGARCH (Normal Distribution)	0.0517	0.1735	0.0523	0.0411	0.0517	0.3784	0.5520
EGARCH (Student t-distribution)	0.0613	7.0798	0.0623	0.0462	0.0613	0.7840	7.8638
EGARCH (Skewed t-distribution)	0.0606	6.2429	0.0618	0.0409	0.0606	1.3742	7.6171
GJR (Normal Distribution)	0.0602	5.8431	0.0622	0.0294	0.0602	3.6593	9.5024
GJR (Student t-distribution)	0.0645	11.4482	0.0666	0.0330	0.0645	3.8141	15.2623
GJR (Skewed t-distribution)	0.0567	2.5450	0.0586	0.0250	0.0567	3.9356	6.4806
Monte Carlo (Normal Distribution)	0.2621	N.a.N.	0.2430	0.3162	0.2622	N.a.N.	N.a.N.
Monte Carlo (Student t-distribution)	0.0535	0.7081	0.0483	0.1457	0.0535	30.8113	31.5195
APARCH (Normal Distribution)	0.0627	8.9010	0.0646	0.0339	0.0627	3.1363	12.0373
APARCH (Student t-distribution)	0.0645	11.4482	0.0666	0.0330	0.0645	3.8141	15.2623
APARCH (Skewed t-distribution)	0.0560	2.0420	0.0574	0.0316	0.0560	2.1864	4.2284

Testing for a 95 percent significance level allows us to reject every model that has test statistics greater than 3.841, or 5.991 in case of the joint test. The numbers marked in bold then represent the models that were not rejected.

For the Kupiec's test, these were: the historical simulation method for all estimation windows, the GARCH model with the normal distribution, the IGARCH model in combination with the skewed t-distribution, the FIGARCH model also based on the skewed t-distribution, the EGARCH model with the normal distribution, and the Skewed-APARCH model.

Testing for independence did, however, reject the historical simulation method for all estimation windows and accepted all GARCH-type models with the exception of the GJR model.

When focusing on the joint test and therefore on the combination of both tests, the data appeared to suggest that the GARCH, IGARCH, and FIGARCH models based on the normal and skewed t-distribution, the EGARCH model with the normal distribution, and the Skewed APARCH cannot be rejected.

Survivors of all three tests, i.e. the models accepted by all three tests, are therefore: the GARCH and EGARCH model based on the normal distribution as well as the IGARCH and FIGARCH model based on the skewed t-distribution and the Skewed-APARCH model.

4.2 Analysis of the period from 2003 to 2007.

The summary statistics of the period from 2003 to 2007 (Table 4) clearly indicated that this period can be described as a period of stability where return distributions were nearer to a normal distribution. Volatility was lower and mean returns slightly higher compared to period that followed.

Computing the absolute differences of the failure rates to the target rates as well as the absolute difference of the proportion of consecutive and non-consecutive violations therefore produced different results than those found for the overall sample.

Table 7 shows the ranking for the different approaches with the same procedure as in the previous subsection but with failure rates and independence indicators generated by the value at risk data estimated on the period from 2003 to 2007.

Table 7: Ranking (2003-2007)

Ranking Based on Absolute Differences (2003-2007)			
	Failure Rate Ranking	Independence Ranking	Avg. Rank
HS (100)	8	23	15.5
HS (250)	6	24	15
HS (500)	11	22	16.5
GARCH (Normal Distribution)	12	13	12.5
GARCH (Student t-distribution)	21	11	16
GARCH (Skewed t-distribution)	7	18	12.5
RiskMetrics	13	2	7.5
IGARCH (Normal Distribution)	13	2	7.5
IGARCH (Student t-distribution)	22	4	13
IGARCH (Skewed t-distribution)	10	1	5.5
FIGARCH (Normal Distribution)	19	5	12
FIGARCH (Student t-distribution)	22	8	15
FIGARCH (Skewed t-distribution)	18	6	12
EGARCH (Normal Distribution)	5	7	6
EGARCH (Student t-distribution)	16	16	16
EGARCH (Skewed t-distribution)	4	14	9
GJR (Normal Distribution)	9	19	14
GJR (Student t-distribution)	20	20	20
GJR (Skewed t-distribution)	1	10	5.5
Monte Carlo (Normal Distribution)	24	15	19.5
Monte Carlo (Student t-distribution)	2	9	5.5
APARCH (Normal Distribution)	13	21	17
APARCH (Student t-distribution)	16	16	16
APARCH (Skewed t-distribution)	3	12	7.5

As shown in the table, the top three according to the failure rate ranking were: the GJR model in combination with a skewed t-distribution, the Monte Carlo simulation based on a student t distributed random number, and the APARCH model assuming a skewed t-distribution. Compared to the period from 2003 to 2013, the only approach which was featured in the top three again was the Monte Carlo simulation. Furthermore, it is worth pointing out that these findings lend support to the student t or skewed t-distribution but not to the normal distribution.

Turning now to the experimental evidence on independence, Table 7 shows that all IGARCH models (normal, student t and skewed t-distribution) and the RiskMetrics model, which itself can be seen as an IGARCH model, represent the top four, since the RiskMetrics and the IGARCH model based on the normal distribution share rank two. This, however, is not very

surprising when investigating the parameters estimated for the IGARCH model. In this case, beta has a value of 0.939930, which is very close to the lambda of 0.94 used by the RiskMetrics approach (see Appendix for other parameter estimations). Moreover, the sample mean return is relatively close to zero, which makes the Normal-IGARCH model nearly equivalent to the RiskMetrics model.

Applying the unweighted average on both rankings then depicts the Monte Carlo simulation (Student t-distribution), Normal-EGARCH, Skewed-IGARCH, and the Skewed-GJR model.

Moreover, it can be seen that the range between the highest average rank and the lowest average rank is closer than the one for the period from 2003 to 2013, which indicates that the differences between the models are not great and that, additionally, some might even share the same rank.

To evaluate the significance of the observations made in Table 7, the test statistics LR_{uc} , LR_{ind} and LR_{cc} , as reported in Table 8, have to be analyzed again in more detail.

At a first glance, it has been found that the number of accepted models is much higher for the period from 2003 to 2007 than for the full sample. The Kupiec's test alone accepts twelve different models. These include: all historical simulation methods, the Monte Carlo simulation based on the student t-distribution, the Skewed IGARCH model, the EGARCH model in combination with the normal distribution and the skewed t-distribution, all GJR models, and, finally, the Skewed-APARCH model.

Testing for independence accepts ten additional models and only rejects the historical simulation methods with an estimation timeframe of 100 and 250 days. The data gathered in this study thus suggests that clusters of violations occur less frequently in this period than compared to the whole sample.

The models that passed all three tests are also the models who passed the Kupiec test, with the exception of the historical simulations.

Table 8: Test Statistics (2003-2007)

Test Statistics for the Selected VaR-Approaches (2003-2007)

<i>Methods</i>	<i>Failure</i>		π_0	π_1	π	LR_{ind}	LR_{cc}
	<i>Rate</i>	LR_{uc}					
HS (100)	0.0418	1.9344	0.0380	0.1296	0.0418	7.2687	9.2031
HS (250)	0.0433	1.2595	0.0389	0.1429	0.0434	9.1561	10.4156
HS (500)	0.0387	3.7538	0.0363	0.1000	0.0387	3.7656	7.5194
GARCH (Normal Distribution)	0.0619	3.6040	0.0635	0.0375	0.0619	1.0023	4.6063
GARCH (Student t-distribution)	0.0689	8.7234	0.0707	0.0449	0.0689	0.9581	9.6815
GARCH (Skewed t-distribution)	0.0573	1.3781	0.0591	0.0270	0.0573	1.6191	2.9972
RiskMetrics	0.0635	4.5626	0.0636	0.0610	0.0635	0.0093	4.5719
IGARCH (Normal Distribution)	0.0635	4.5626	0.0636	0.0610	0.0635	0.0093	4.5719
IGARCH (Student t-distribution)	0.0774	17.6221	0.0772	0.0800	0.0774	0.0102	17.6323
IGARCH (Skewed t-distribution)	0.0611	3.1645	0.0610	0.0633	0.0611	0.0067	3.1712
FIGARCH (Normal Distribution)	0.0666	6.7902	0.0663	0.0698	0.0666	0.0150	6.8052
FIGARCH (Student t-distribution)	0.0774	17.6221	0.0763	0.0900	0.0774	0.2309	17.8530
FIGARCH (Skewed t-distribution)	0.0658	6.1952	0.0663	0.0588	0.0658	0.0742	6.2694
EGARCH (Normal Distribution)	0.0550	0.6475	0.0557	0.0423	0.0550	0.2514	0.8990
EGARCH (Student t-distribution)	0.0642	5.0812	0.0662	0.0361	0.0642	1.3568	6.4380
EGARCH (Skewed t-distribution)	0.0542	0.4631	0.0556	0.0286	0.0542	1.1245	1.5876
GJR (Normal Distribution)	0.0588	2.0091	0.0609	0.0263	0.0588	1.8978	3.9068
GJR (Student t-distribution)	0.0681	8.0545	0.0706	0.0341	0.0681	2.0589	10.1134
GJR (Skewed t-distribution)	0.0519	0.0928	0.0531	0.0299	0.0519	0.8095	0.9023
Monte Carlo (Normal Distribution)	0.2546	N.a.N	0.2471	0.2766	0.2546	N.a.N	N.a.N
Monte Carlo (Student t-distribution)	0.0480	0.1116	0.0472	0.0645	0.0480	0.3549	0.4665
APARCH (Normal Distribution)	0.0635	4.5626	0.0661	0.0244	0.0635	2.8561	7.4188
APARCH (Student t-distribution)	0.0642	5.0812	0.0662	0.0361	0.0642	1.3568	6.4380
APARCH (Skewed t-distribution)	0.0534	0.3089	0.0548	0.0290	0.0534	1.0141	1.3230

4.3 Analysis of the period from 2008 to 2013

Having described the test results for the overall sample and the pre-crisis period, this part will now consider the crucial period from 2008 to 2013, which includes the financial crisis. With its epicenter in the collapse of the investment bank Lehman Brothers at the end of September 2008, the financial crisis caused disturbance and panic in stock markets worldwide. These effects did not spare the German stock market and hence were noticeable in the sub-sample at hand: volatility rose dramatically while the mean return decreased and skewness turned from negative to positive. The return distribution got longer tails and a bigger excess kurtosis (see 3.6 Data Collection, Editing, Coding and Analysis). Bearing this mind, the ranking appears to be of special interest.

Table 9: Ranking (2008-2013)

Ranking Based on Absolute Differences (2008-2013)			
	Failure Rate Ranking	Independence Ranking	Avg. Rank
HS (100)	9	21	15
HS (250)	4	22	13
HS (500)	3	24	13.5
GARCH (Normal Distribution)	8	3	5.5
GARCH (Student t-distribution)	21	1	11
GARCH (Skewed t-distribution)	11	8	9.5
RiskMetrics	18	2	10
IGARCH (Normal Distribution)	11	8	9.5
IGARCH (Student t-distribution)	22	13	17.5
IGARCH (Skewed t-distribution)	6	6	6
FIGARCH (Normal Distribution)	13	5	9
FIGARCH (Student t-distribution)	23	11	17
FIGARCH (Skewed t-distribution)	10	12	11
EGARCH (Normal Distribution)	1	4	2.5
EGARCH (Student t-distribution)	16	7	11.5
EGARCH (Skewed t-distribution)	15	10	12.5
GJR (Normal Distribution)	14	16	15
GJR (Student t-distribution)	19	18	18.5
GJR (Skewed t-distribution)	6	17	11.5
Monte Carlo (Normal Distribution)	24	20	22
Monte Carlo (Student t-distribution)	2	23	12.5
APARCH (Normal Distribution)	17	15	16
APARCH (Student t-distribution)	19	18	18.5
APARCH (Skewed t-distribution)	5	14	9.5

Again, the historical simulation with a rolling estimation window of 500 days is featured in the top three considering the failure rate together with the EGARCH model based on a

skewed t-distribution and another historical simulation method using an estimation timeframe of 250 days.

Focusing on the absolute difference of the proportion of consecutive and non-consecutive violations, an overlap can be found to the ranking based on the failure rate, namely the EGARCH model in combination with the skewed t-distribution. The other two models include the EGARCH and IGARCH model with a student t-distribution.

On average, the two highest ranks are occupied by two EGARCH models, one with a skewed t-distribution and one with a student t-distribution. The next lower rank is then represented by the Normal-EGARCH.

From these findings, it can be seen that it is difficult to come to a clear and distinctive statement. Hence, the Kupiec's, Christoffersen's, and joint test shall be consulted. The data generated by these is reported in Table 10.

Again, all historical simulation approaches are accepted by the Kupiec test but rejected by the Christoffersen test and joint test. All representatives of the EGARCH family have failure rates not significantly different from the VaR alpha. Additionally, the EGARCH model in combination with all three distributions is accepted when testing for independence and is consequently not rejected by the joint test. Similarly, the APARCH model with a skewed t-distribution did pass the Kupiec test as well as the joint test. Finally, only the GJR model based on a skewed t-distribution and the Monte Carlo simulation with student t-distributed random numbers as a basis are accepted by the Kupiec test, but only the GJR-GARCH model is not rejected by the joint test and Christoffersen test.

In summary, only the EGARCH models and the Skewed-GJR model as well as the Skewed-APARCH appear to suit all demanded properties during the period from 2008 to 2013.

Table 10: Test Statistics (2008-2013)

Test Statistics for the Selected VaR-Approaches (2008-2013)							
<i>Methods</i>	<i>Failure</i>		π_0	π_1	π	LR_{ind}	LR_{cc}
	<i>Rate</i>	LR_{uc}					
HS (100)	0.0431	1.6021	0.0403	0.1061	0.0431	4.8656	6.4678
HS (250)	0.0451	0.8095	0.0410	0.1304	0.0451	8.4587	9.2682
HS (500)	0.0503	0.0028	0.0426	0.1948	0.0503	22.0307	22.0335
GARCH (Normal Distribution)	0.0647	6.3691	0.0656	0.0505	0.0647	0.3758	6.7449
GARCH (Student t-distribution)	0.0666	8.1018	0.0679	0.0490	0.0666	0.5925	8.6943
GARCH (Skewed t-distribution)	0.0640	5.8345	0.0649	0.0510	0.0640	0.3142	6.1487
RiskMetrics	0.0653	6.9253	0.0643	0.0800	0.0653	0.3558	7.2811
IGARCH (Normal Distribution)	0.0640	5.8345	0.0649	0.0510	0.0640	0.3142	6.1487
IGARCH (Student t-distribution)	0.0732	15.2154	0.0733	0.0714	0.0732	0.0053	15.2208
IGARCH (Skewed t-distribution)	0.0640	5.8345	0.0649	0.0510	0.0640	0.3142	6.1487
FIGARCH (Normal Distribution)	0.0712	12.8715	0.0689	0.1009	0.0712	1.4134	14.2848
FIGARCH (Student t-distribution)	0.0758	18.6097	0.0721	0.1207	0.0758	3.1493	21.7590
FIGARCH (Skewed t-distribution)	0.0660	7.5030	0.0615	0.1287	0.0660	5.6051	13.1080
EGARCH (Normal Distribution)	0.0588	2.3606	0.0597	0.0444	0.0588	0.3845	2.7451
EGARCH (Student t-distribution)	0.0425	1.9291	0.0430	0.0308	0.0425	0.2503	2.1794
EGARCH (Skewed t-distribution)	0.0536	0.3996	0.0538	0.0488	0.0536	0.0401	0.4397
GJR (Normal Distribution)	0.0614	3.9161	0.0626	0.0426	0.0614	0.6828	4.5989
GJR (Student t-distribution)	0.0653	6.9253	0.0671	0.0400	0.0653	1.2811	8.2065
GJR (Skewed t-distribution)	0.0594	2.7146	0.0618	0.0220	0.0594	3.1148	5.8294
Monte Carlo (Normal Distribution)	0.2698	N.a.N.	0.2424	0.3438	0.2698	N.a.N.	N.a.N.
Monte Carlo (Student t-distribution)	0.0594	2.7146	0.0514	0.1868	0.0594	19.1441	21.8587
APARCH (Normal Distribution)	0.0614	3.9161	0.0633	0.0319	0.0614	1.7950	5.7111
APARCH (Student t-distribution)	0.0666	8.1018	0.0693	0.0294	0.0666	2.9947	11.0965
APARCH (Skewed t-distribution)	0.0562	1.1829	0.0581	0.0233	0.0562	2.3353	3.5181

5. Discussion

This part of the thesis discusses in summary the findings which emerged from the statistical analysis presented in the previous section and points out the major limitations of this research. The final chapter of this thesis is therefore divided into two parts: Discussion and Research Limitations.

5.1 Discussion

The overall aim of the thesis is to identify and rank the most accurate approaches to determine value at risk in normal and extreme conditions. To meet these requirements, the hypothesis introduced in part 3.1 can be evaluated with the help of the information provided in the previous chapter.

In summary, the tests applied to the value at risk data yielded the following approaches as significantly accurate.

Table 11: Ranking Overview

Significant Accurate VaR-Approaches		
2003 - 2013	2003 - 2007	2008 - 2013
EGARCH (Normal Distribution)	GJR (Skewed t-distribution)*	EGARCH (Normal Distribution)
GARCH (Normal Distribution)	IGARCH (Skewed t-distribution)*	APARCH (Skewed t-distribution)
IGARCH (Skewed t-distribution)	Monte Carlo (Student t-distribution)*	EGARCH (Student t-distribution)*
APARCH (Skewed t-distribution)	EGARCH (Normal Distribution)	GJR (Skewed t-distribution)*
FIGARCH (Skewed t-distribution)	APARCH (Skewed t-distribution)	EGARCH (Skewed t-distribution)*
	EGARCH (Skewed t-distribution)	
	GARCH (Normal Distribution)**	
	GARCH (Skewed t-distribution)**	
	GJR (Normal Distribution)	

Sorting these approaches with regard to their average ranking based on absolute differences then leads to three individual rankings as presented in Table 11. 0. Approaches, which share the same place, are marked with * and **.

Considering only the period from 2008 to 2013, it is apparent that all the approaches which passed the tests are models that account for the asymmetry. This indicates the presence of the leverage effect in the CDAX returns, which gives evidence for a negative correlation between stock returns and volatility as proposed by Black (1976), Bentes et al. (2013b), and Tao & Green (2012) (see 2.2.6 GJR-GARCH, 2.2.7 APARCH and 2.2.8 EGARCH). Since the APARCH model was accepted, the long memory property might play a role as well. It thus can be concluded that, in times of extreme market condition, asymmetric models outperform symmetric models like the GARCH model when applied to the German market.

This conclusion is in line with the findings of Awartani & Corradi (2005), Diamandis et al. (2011), and especially Mokni et al. (2009) and Curto & Pinto (2012), who investigate the effects of the financial crisis on volatility processes (see 2.2.6 GJR-GARCH and 2.2.7 APARCH).

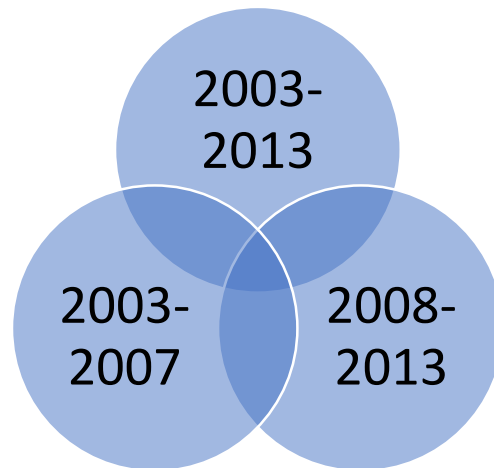
Focusing on the pre-crisis period clearly shows that more models are accepted than in the period from 2008 to 2013. Again, nearly all models are models accommodating for the leverage effect. Moreover, it can be seen that, additionally, the IGARCH model is featured in the top three. The long memory effect is thus present in this period as well. Further, it is more difficult to come to a definite statement about the ranking in this time since several approaches share the same place.

Another interesting fact is that non-parametric approaches represented by the historical simulation are not part of any ranking displayed in Table 11. Taking a closer look at the Table 6, Table 8 and Table 10, reveals that, although accepted by the Kupiec test, the historical method is always rejected by the Christoffersen test. This implies that the historical simulation lacks the dynamics to account for volatility clustering and hence to generate independent violations. This, however, is not surprising, as it has already been observed by Gaglianone et al. (2011) and Perignon & Smith (2008) (see 2.2.1 Historical Simulation).

Moreover, it can be seen that models based on the skewed t-distribution are rejected far less often than those based on a student t or normal distribution.

In the next step, determining whether these approaches are accurate in normal and extreme market conditions, the approaches tested as significantly accurate for only one or two samples will be eliminated.

Figure 14: VaR Intersections



The remaining approaches to value at risk then form the intersection of all samples, as depicted by Figure 14. These are the EGARCH models in combination with the normal distribution and on the basis of the skewed t-distribution and the Skewed-APARCH model. Again, it can be seen that asymmetric models outperform symmetric models.

5.2 Research Limitations and Constrains

So far, this section has focused on the findings and their implications. It is now necessary to explain the limitations and constraints that restrict the validity and reliability of the research conducted.

To keep the focus of this study within boundaries, several simplifications were made during the process of completing said study; it is important to be aware of these when interpreting the findings.

As described earlier, value at risk does have many adjusting screws. Ranging from the time horizon, over the confidence level and the choice of parameter estimation models to the distribution assumptions, these factors offer research possibilities which focus on each individually.

To ensure the quality of this work for the given time, this thesis focused on the choice of parameter estimation models and assumed everything else to be constant. Therefore, a framework was chosen which sets the confidence level of the value at risk at 95 percent and the time horizon to one day. The estimation was conducted on the basis of the daily returns of the German CDAX, which might not allow for simply transferring the findings to other assets.

Moreover, emphasis was put on a single underlying asset and not on a portfolio of multiple assets, which made the estimation of correlations and covariance redundant. This, however, is again possible with a wide range of additional models.

Next to this, the assumptions about the probability distribution of the underlying returns was limited to the three major distributions and ignored other distributions supported by the literature, such as the normal-inverse Gaussian distribution or the general error distribution.

Additionally, the application of tests was limited to the most-used test (Kupiec and Christoffersen) and disregarded other tests, which focus on certain properties of value at risk and might come to a different conclusion.

Finally, one of the biggest constraints introduced is the choice of the back testing period. To structure the findings and to avoid confusion and complexity, in-sample back testing was applied, which means that the period that was used to estimate the coefficients of the GARCH models is the same period for which the approaches were back tested. Out-of-sample forecasts were not taken into consideration.

6. Conclusion

The rapidly growing literature on value at risk indicates the vast interest in as well as the relevancy of the topic. Review of current empirical studies, however, has shown that there is no consensus on which approach is most suitable in practice. For each model, a great amount of research can be found that supports that respective model. Depending on the underlying assets, different approaches can be identified as most suitable. However, even then, an immense number of models are championed for whole-asset classes. This problematic nature is then fueled by the development of new models or deviation of existing models. The number of non-parametric, parametric, and semi-parametric modes is steadily growing, with each model focusing on different properties of financial time series such as long memory or leverage effects.

This thesis aimed to identify the approaches with the most impact and to explain them in greater detail. Hence, models of each category were chosen and compared. The non-parametric models were represented by the historical simulation, the parametric models by GARCH-type models (GARCH, RiskMetrics, IGARCH, FIGARCH, GJR, APARCH and EGARCH), and the semi-parametric models by the Monte Carlo simulation. The functional principle of each approach was explained, compared, and contrasted.

The next step was to estimate value at risk for one sample ranging from 2003 to 2013 and two sub-samples capturing the periods from 2003 to 2007 and 2008 to 2013. This way, a comparison of pre-crisis and crisis condition was possible.

Using the data generated by this, a ranking for each approach in each sample was conducted on the basis of absolute differences. The highest rank in the overall sample was represented by the Normal-EGARCH model, the highest in the pre-crisis period is found to be the Skewed-GJR model together with the Skewed-IGARCH model and the Monte Carlo simulation based on the student t-distribution, and the highest rank during the crisis was again held by the Normal-EGARCH model.

After that, three widely used statistical tests, namely, the Kupiec, Christoffersen, and Joint tests, were applied to examine the accuracy of the selected approaches. Approaches failing these tests were then no longer considered.

By doing this it came apparent, that models accounting for asymmetries and long memory were accepted most frequently. The approaches found to survive each test in each period were the EGARCH model in combination with the normal distribution and on basis of the skewed t-distribution as well as the Skewed-APARCH model.

Based on the in-sample back test applied to the CDAX returns for the given period, it can be concluded that these models outperform the other selected models.

Nevertheless, considering the research limitations and constraints imposed to this thesis, further research is suggested to take into account multiple confidence levels, time horizons, and a greater number of underlying assets mapping different markets worldwide. Furthermore, other back-testing approaches and out-of-sample back testing should be applied to create comparability and come to robust conclusions.

Publication bibliography

- Acerbi, C.; Tasche, D. (2002): Expected shortfall: A natural coherent alternative to value at risk. In *Economic Notes* 31 (2), pp. 379–388. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,cookie,url,athens&db=bth&AN=6832302&site=eds-live>.
- Aczel, Amir D.; Sounderpandian, Jayavel (2009): *Complete Business Statistics. 7th ed.* Boston: McGraw-Hill/Irwin (The McGraw-Hill/Irwin series operations and decision sciences).
- Alexander, C. (2009): *Market Risk Analysis, Value at Risk Models*. Wiley. Available at <http://books.google.ie/books?id=e03cHzItv0AC>.
- Angelidis, Timotheos; Benos, Alexandros; Degiannakis, Stavros (2004): The use of GARCH models in VaR estimation. In *Statistical Methodology* 1 (1), pp. 105–128. Available at <http://economics.soc.uoc.gr/macro/8conf/docs/The%20Use%20of%20GARCH%20Models%20in%20VaR%20Estimation.pdf>.
- Angelidis, Timotheos; Skiadopoulos, George (2008): Measuring the market risk of freight rates: A value-at-risk approach. In *International Journal of Theoretical and Applied Finance* 11 (05), pp. 447–469. Available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=988739.
- Awartani, Basel; Corradi, Valentina (2005): Predicting the volatility of the S&P-500 stock index via GARCH models: The role of asymmetries. In *International Journal of Forecasting* 21 (1), pp. 167–183. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=16014780&site=eds-live>.
- Baillie, Richard T.; Bollerslev, Tim; Mikkelsen, Hans Ole (1996): Fractionally integrated generalized autoregressive conditional heteroscedasticity. In *Journal of Econometrics* 74 (1), pp. 3–30.
- Baillie, Richard T.; DeGennaro, Ramon P. (1990): Stock returns and volatility. In *Journal of Financial and Quantitative Analysis* 25 (2), pp. 203–214. Available at <https://www.msu.edu/user/baillie/JFQA.1990.pdf>.
- Baillie, Richard T.; Morana, Claudio (2009): Modelling long memory and structural breaks in conditional variances: An adaptive i FIGARCH approach. In *Journal of Economic Dynamics and Control* 33 (8), pp. 1577–1592.
- Bauwens, Luc; Hafner, Christian; Laurent, Sébastien (2012): Volatility Models. In Luc Bauwens, Christian Hafner, Sébastien Laurent (Eds.): *Handbook of volatility models and their applications*. Hoboken, N.J: Wiley (Wiley handbooks in financial engineering and econometrics), pp. 1–45.
- Bentes, Sónia R.; Menezes, Rui; Ferreira, Nuno B. (2013a): On the asymmetric behaviour of stock market volatility: Evidence from three countries. In *International Journal of Academic Research* 5 (4), pp. 24–32. Available at

<http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=a9h&AN=92947455&site=eds-live>.

Available at Black, F. (1976): Studies of stock market volatility changes. In *Proceedings of the American Statistical Association*.

Bollerslev, Tim (1986): Generalized autoregressive conditional heteroscedasticity. In *Journal of Econometrics* 31 (3), pp. 307–327. DOI: 10.1016/0304-4076(86)90063-1.

Bollerslev, Tim (1987): A conditionally heteroskedastic time series model for speculative prices and rates of return. In *The Review of Economics and Statistics*, pp. 542–547. Available at <http://www.hss.caltech.edu/~camerer/SS280/BollerslevRES87.pdf>.

Brooks, Chris; Persaud, Gita (2003): The Effect of Asymmetries on Stock Index Return Value-at-Risk Estimates. In *Journal of Risk Finance (Euromoney Institutional Investor PLC)* 4 (2), pp. 29–42. Available at

<http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=9641250&site=eds-live>.

Bryman, Alan (2008): *Social Research Methods. 3rd ed.* Oxford, New York: Oxford University Press.

Carnero, M. Angeles; Peña, Daniel; Ruiz, Esther (2004): Persistence and kurtosis in GARCH and stochastic volatility models. In *Journal of Financial Econometrics* 2 (2), pp. 319–342. Available at http://e-archivo.uc3m.es/bitstream/handle/10016/4856/persistence_ruiz_JFE_2004_ps.pdf;jsessionid=1206417CBD44A24DCCE8FFF4BD6C8493?sequence=1.

Christie, Andrew A. (1982): The stochastic behavior of common stock variances: Value, leverage and interest rate effects. In *Journal of Financial Economics* 10 (4), pp. 407–432.

Cochran, Steven J.; Mansur, Iqbal; Odusami, Babatunde (2012): Volatility persistence in metal returns: A FIGARCH approach. In *Journal of Economics and Business* 64 (4), pp. 287–305. DOI: 10.1016/j.jeconbus.2012.03.001.

Cummins, Clint; Doornik, Jürgen A.; Hendry, David F.; Koopman, Siem Jan; Laurent, Sébastien; Ooms, Marius (2014): OxMetrics 7. G@RCH 7. Version 7: Timberlake Consultants. Available at <http://www.doornik.com/oxmetrics.html>, checked on 6/27/2014.

Curto, José Dias; Pinto, José Castro (2012): Predicting the financial crisis volatility. In *Economic Computation & Economic Cybernetics Studies & Research* 46 (1), pp. 183–195. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=78951349&site=eds-live>.

Dedu, Silvia; Fulga, Cristina (2011): Value-at-risk estimation comparative approach with applications to optimization problems. In *Economic Computation & Economic Cybernetics Studies & Research* 45 (1), pp. 1–16. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=65048682&site=eds-live>.

- Degiannakis, Stavros; Floros, Christos; Dent, Pamela (2013): Forecasting value-at-risk and expected shortfall using fractionally integrated models of conditional volatility: International evidence. In *International Review of Financial Analysis* 27 (0), pp. 21–33. DOI: 10.1016/j.irfa.2012.06.001.
- Diamandis, Panayiotis F.; Drakos, Anastassios A.; Kouretas, Georgios P.; Zarangas, Leonidas (2011): Value-at-risk for long and short trading positions: Evidence from developed and emerging equity markets. In *International Review of Financial Analysis* 20 (3), pp. 165–176. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=60665589&site=eds-live>.
- Ding, Zhuangxin; Granger, Clive W. J.; Engle, Robert F. (1993): A long memory property of stock market returns and a new model. In *Journal of Empirical Finance* 1 (1), pp. 83–106. Available at [http://www.netgrate.com/index_files/Research%20Library/Catalogue/Quantitative%20Analysis/Long-Range%20Dependence/A%20Long%20memory%20property%20of%20Stock%20Returns%20and%20a%20new%20Model\(Ding,Granger%20and%20Engle\).pdf](http://www.netgrate.com/index_files/Research%20Library/Catalogue/Quantitative%20Analysis/Long-Range%20Dependence/A%20Long%20memory%20property%20of%20Stock%20Returns%20and%20a%20new%20Model(Ding,Granger%20and%20Engle).pdf).
- Emmer, Susanne; Kratz, Marie; Tasche, Dirk (2013): What is the best risk measure in practice? A comparison of standard measures. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,cookie,url,athens&db=edsarx&AN=1312.1645&site=eds-live>.
- Engle, Robert F. (1982): Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. In *Econometrica: Journal of the Econometric Society*, pp. 987–1007.
- Engle, Robert F.; Bollerslev, Tim (1986): Modelling the persistence of conditional variances. In *Econometric reviews* 5 (1), pp. 1–50. Available at http://public.econ.duke.edu/~boller/Published_Papers/ectrev_86.pdf.
- Fabozzi, Frank J.; Stoyanov, Stoyan V.; Rachev, Svetlozar T. (2013): Computational aspects of portfolio risk estimation in volatile markets: a survey. In *Studies in Nonlinear Dynamics & Econometrics* 17 (1), pp. 103–120. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=85597376&site=eds-live>.
- Fama, Eugene F. (1965): The behavior of stock-market prices. In *Journal of Business*, pp. 34–105. Available at http://pindarwin.blog.com/files/2009/11/the_behavior_of_stock_market_prices.pdf.
- Ferreira, Nuno B.; Menezes, Rui; Mendes, Diana A. (2007): Asymmetric conditional volatility in international stock markets. In *Physica A: Statistical Mechanics and its Applications* 382 (1), pp. 73–80.
- Füss, Roland; Adams, Zeno; Kaiser, Dieter G. (2010): The predictive power of value-at-risk models in commodity futures markets. In *Journal of Asset Management* 11 (4), pp. 261–285. Available at

<http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=54301349&site=eds-live>.

Gaglianone, Wagner Piazza; Lima, Luiz Renato; Linton, Oliver; Smith, Daniel R. (2011): Evaluating value-at-risk models via quantile regression. In *Journal of Business & Economic Statistics* 29 (1). Available at <http://www.eea-esem.com/files/papers/EEA-ESEM/2008/21/Evaluating%20VaR%20models.pdf>.

Giot, Pierre; Laurent, Sébastien (2003a): Market risk in commodity markets: a VaR approach. In *Energy Economics* 25 (5), pp. 435–457. Available at <http://webdoc.sub.gwdg.de/ebook/serien/e/CORE/dp2003-28.pdf>.

Giot, Pierre; Laurent, Sébastien (2003b): Value-at-risk for long and short trading positions. In *Journal of Applied Econometrics* 18 (6), pp. 641–664. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=11992286&site=eds-live>.

Glosten, Lawrence R.; Jagannathan, Ravi; Runkle, David E. (1993): On the relation between the expected value and the volatility of the nominal excess return on stocks. In *The Journal of Finance* 48 (5), pp. 1779–1801. Available at <http://finance.martinsewell.com/stylized-facts/volatility/GlostenJagannathanRunkle1993.pdf>.

Hamilton, James D.; Susmel, Raul (1994): Autoregressive conditional heteroscedasticity and changes in regime. In *Journal of Econometrics* 64 (1), pp. 307–333. Available at http://schwert.ssb.rochester.edu/f532/je94_hs.pdf.

Hansen, Peter R.; Lunde, Asger (2005): A forecast comparison of volatility models: Does anything beat a GARCH(1,1)? In *Journal of Applied Econometrics* 20 (7), pp. 873–889. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=19240397&site=eds-live>.

Harvey, A. C. (1993): *Time Series Models*. 2nd ed. Cambridge, Mass: MIT Press.

Hendricks, Darryll (1996): Evaluation of value-at-risk models using historical data. In *Federal Reserve Bank of New York Economic Policy Review* 2 (1), pp. 39–69. Available at <http://www.nyfedeconomists.org/research/epr/96v02n1/9604hend.pdf>, checked on 6/23/2014.

Hull, John (2012): *Risk Management and Financial Institutions*. 3rd ed. Hoboken, N.J: John Wiley & Sons (Wiley Finance).

Hull, John C. (2011): *Options, Futures, and Other Derivatives*. Harlow: Pearson Education.

Hung, Jui-Cheng; Lee, Ming-Chih; Liu, Hung-Chun (2008): Estimation of value-at-risk for energy commodities via fat-tailed GARCH models. In *Energy Economics* 30 (3), pp. 1173–1191. Available at <http://edoc.ypu.edu.tw:8080/paper/finance/%E8%B2%A1%E9%87%91%E7%B3%BB-%E6%B4%AA%E7%91%9E%E6%88%90/Estimation%20of%20Value-at-Risk%20for%20energy%20commodities%20via%20fat-tailed%20garch%20models-EE-2008.pdf>.

- J.P. Morgan (1996): RiskMetrics. Technical Document. With assistance of Reuters Ltd. New York. Available at http://pascal.iseg.utl.pt/~aafonso/eif/rm/TD4ePt_2.pdf.
- Jarque, Carlos M.; Bera, Anil K. (1987): A test for normality of observations and regression residuals. In *International Statistical Review/Revue Internationale de Statistique*, pp. 163–172.
- Jorion, Philippe (2007a): *Value at Risk: The New Benchmark for Managing Financial Risk*. 3rd ed. New York: McGraw-Hill.
- Kearns, Phillip; Pagan, Adrian (1997): Estimating the density tail index for financial time series. In *Review of Economics & Statistics* 79 (2), pp. 171–175. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=9708083505&site=eds-live>.
- Lambert, Philippe; Laurent, S. (2001): Modelling financial time series using GARCH-type models with a skewed Student-distribution for the innovations.
- Lamoureux, Christopher G.; Lastrapes, William D. (1990): Persistence in variance, structural change, and the GARCH model. In *Journal of Business & Economic Statistics* 8 (2), pp. 225–234. Available at http://people.terry.uga.edu/last/papers/jbes_1990.pdf.
- Laurent, Sébastien (2014): GARCH: OxMetrics 7. G@RCH 7. Version 7: Timberlake Consultants. Available at <http://www.timberlake.co.uk/slaurant/G@RCH/default.htm>, checked on 6/27/2014.
- Lee, Cheng-Few; Su, Jung-Bin (2012): Alternative statistical distributions for estimating value-at-risk: Theory and evidence. In *Review of Quantitative Finance & Accounting* 39 (3), pp. 309–331. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=80039771&site=eds-live>.
- Liu, Hung-Chun; Hung, Jui-Cheng (2010): Forecasting S&P-100 stock index volatility: The role of volatility asymmetry and distributional assumption in GARCH models. In *Expert Systems with Applications* 37 (7), pp. 4928–4934. DOI: 10.1016/j.eswa.2009.12.022.
- Mandelbrot, Benoit (1963): The variation of certain speculative prices. In *The Journal of Business* 36 (4), pp. 394–419. Available at http://web.williams.edu/Mathematics/sjmillier/public_html/341Fa09/econ/Mandelbrot_VariationCertainSpeculativePrices.pdf.
- Manganelli, Simone; Engle, Robert F. (2001): Value at risk models in finance. European Central Bank. Frankfurt am Main. Available at <http://exinfmtvs.securesites.net/training/pdffiles/variskbiz.pdf>.
- MOHAMMAD, Faiz; ANSARI, Valeed A. (2013): Empirical distributions of nifty returns from 1991 to 2010. In *Hyperion International Journal of Econophysics & New Economy* 6 (2), pp. 265–273. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=a9h&AN=95057237&site=eds-live>.

- Mokni, Khaled; Mighri, Zouheir; Mansouri, Faysal (2009): On the effect of subprime crisis on value-at-risk estimation: GARCH family models approach. In *International Journal of Economics and Finance* 1 (2), pp. P88.
- Morana, C. (2002): IGARCH effects: an interpretation. In *Applied Economics Letters* 9 (11), pp. 745–748. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=7253103&site=eds-live>.
- Nelson, Daniel B. (1991): Conditional heteroscedasticity in asset returns: A new approach. In *Econometrica: Journal of the Econometric Society*, pp. 347–370. Available at <http://www.jstor.org/discover/10.2307/2938260?uid=3738232&uid=2134&uid=376259273&uid=2&uid=70&uid=3&uid=376259263&uid=60&purchase-type=article&accessType=none&sid=21104498540993&showMyJstorPss=false&seq=4&showAccess=false>.
- Nelson, Daniel B.; Cao, Charles Q. (1992): Inequality constraints in the univariate GARCH model. In *Journal of Business & Economic Statistics* 10 (2), pp. 229–235. Available at <http://www.personal.psu.edu/qxc2/research/jbes.pdf>.
- Nikolic-Dorić, Emilija; Doric, Dragan (2011): Dynamic Value at Risk Estimation for BELEX15. In *Advances in Methodology & Statistics / Metodoloski zvezki* 8 (1), pp. 79–98. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=a9h&AN=67735386&site=eds-live>.
- Perignon, Christophe; Smith, Daniel R. (2008): A new approach to comparing VaR estimation methods. In *The Journal of Derivatives* 16 (2), pp. 54–66. Available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=981207.
- Picoult, Evan (2002): Quantifying the risks of trading. In Dempster, M. A. H (Ed.): *Risk management. Value at risk and beyond*. Cambridge, New York: Cambridge University Press, pp. 1–58.
- Po-Cheng Wu; Cheng-Kun Kuo; Chih-Wei Lee (2012): EVALUATION OF MULTI-ASSET VALUE AT RISK: EVIDENCE FROM TAIWAN. In *Global Journal of Business Research (GJBR)* 6 (4), pp. 23–34. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=82211161&site=eds-live>.
- Powell, Robert J.; Yun Hsing Cheung (2012): Anybody can do value at risk: A Nonparametric Teaching Study. In *Australasian Accounting Business & Finance Journal* 6 (1), pp. 111–123. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,cookie,url,athens&db=bth&AN=74159636&site=eds-live>.
- Praetz, Peter D. (1972): The distribution of share price changes. In *The Journal of Business*, 1972 (1), p. 49. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=edsjsr&AN=edsjsr.2351598&site=eds-live>.

- Prepic, A.; Unosson, M. (2014): A comparison of GARCH-class models and MIDAS regression with applications in volatility prediction and value at risk estimation. Available at <http://www.diva-portal.org/smash/get/diva2:723840/FULLTEXT01.pdf>.
- Rejeb, Aymen Ben; Salha, Ousama Ben; Rrjeb, Jaleleddine Ben (2012): Value-at-risk analysis for the Tunisian foreign exchange market: A comparative study. In *Theoretical & Applied Economics* 19 (5), pp. 93–114. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=76131393&site=eds-live>.
- Sarma, Mandira; Thomas, Susan; Shah, Ajay (2003): Selection of value-at-risk models. In *Journal of Forecasting* 22 (4), pp. 337–358. Available at http://202.54.18.156/faculty/susant/PDFDOCS/SarmaThomasShah2002_var_selection.pdf.
- Sironi, Andrea; Resti, Andrea (2007): Risk management and shareholders' value in banking. From risk measurement models to capital allocation policies. Chichester, West Sussex [England], Hoboken, NJ: Wiley (Wiley Finance).
- So, Mike K. P.; Yu, Philip L. H. (2006): Empirical analysis of GARCH models in value at risk estimation. In *Journal of International Financial Markets, Institutions and Money* 16 (2), pp. 180–197.
- Su, Y. C.; Huang, H. C.; Lin, Y. J. (2011): GJR-GARCH model in value-at-risk of financial holdings. In *Applied Financial Economics* 21 (24), pp. 1819–1829. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=65500854&site=eds-live>.
- Tao, Juan; Green, Christopher J. (2012): Asymmetries, causality and correlation between FTSE100 spot and futures: A DCC-TGARCH-M analysis. In *International Review of Financial Analysis* 24, pp. 26–37. DOI: 10.1016/j.irfa.2012.07.002.
- Tayefi, Maryam; Ramanathan, T. V. (2012): An overview of FIGARCH and related time series models. In *Austrian Journal of Statistics [4] Brietzke, EHM, Lopes, Bisognin, A closed formula for the Durbin-Levinson's algorithm in seasonal fractionally integrated processes*.
- Vose, D. (2008): *Risk Analysis: A Quantitative Guide*. John Wiley & Sons. Available at <http://books.google.ie/books?id=9CaoAqaRcVwC>.
- Wong, W. K. (2009): Backtesting the tail risk of VaR in holding US dollar. In *Applied Financial Economics* 19 (4), pp. 327–337. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=36449251&site=eds-live>.
- Yu Chuan Huang; Bor-Jing Lin, Anne (2004): Value-at-risk analysis for Taiwan stock index futures: Fat tails and conditional asymmetries in return innovations. In *Review of Quantitative Finance & Accounting* 22 (2), pp. 79–95. Available at <http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,custuid,cookie,url,athens&custid=s6175963&db=bth&AN=12683520&site=eds-live>.

Appendix A: Reflections on Learning

Self-Appraisal and Process

My goal for my professional life is to found my own financial service firm, which provides asset and wealth management for investors.

To find a suitable topic for my master thesis my strategy thus included to ask friends and acquaintances, who work in the financial service and banking sector, about current problems and challenges. One of the responses to my enquiries addressed the topic of value at risk with regard to capital requirements and regulation of actively managed investment funds. Their special interest here was to find the best way to estimate value at risk without overestimating risk and therefore being punished with higher capital requirements.

My first idea thus was to write about the use of value at risk in active portfolio management with respect to UCITS IV, which is the European regulation for investment funds.

A first preliminary review of literature in this area quickly showed that one of the most important factors is the estimation of volatility. Since the possibilities to do this are numerous, I decided to concentrate on the most academically discussed ones: the GRACH-type models. It came apparent that this alone would give many opportunities to conduct research on. I therefore decided to alter my topic to the evaluation of the various value at risk approaches in normal and extreme market conditions.

The first challenge in writing my thesis was then to develop a suitable strategy to find relevant information to build on. My solution to this problem included the following steps:

First, I read the original articles written by the developers of the various value-at-risk approaches.

Next to this, I had to refresh my statistical and mathematical knowledge about the procedures applied in these articles. By doing this I was able to acquire a lot of new knowledge and skills in this area and could extend my “toolbox” by some widely used models.

Third, I took the original articles as a basis for reviewing further literature. My focus here was set to literature that was concerned with the evaluation of the value at risk models and that was not written before 2000. This way I was able to present a contemporary cross section of empirical studies and opinions on the selected models.

Again, the next final step was to learn about the statistical tests and methods applied by these to evaluate the value at risk approaches. This equipped me with valuable knowledge and tools to conduct my own research later.

Regarding this strategy it can be seen that it was necessary to move outside my discipline to find sufficient sources, i.e. statistics journals and textbooks. Moreover, I realized that finding information on my topic was not difficult because of a small supply of journal articles but because of the vast number of information that can be found about the various value at risk

approaches. The temptation to include all the discovered aspects of value at risk in my thesis was permanently present. I therefore had to develop the skill of sorting out irrelevant information and to utilize the selected information in a timely manner.

Another point worth mentioning is the fact that none of the approaches could be determined as most suitable or best working based on the literature reviewed, since different studies came to some extent to quite different conclusions. This however only supported the relevance of my own research and increased my interest in my chosen topic.

Problem –Solving

The next problem I encountered was the implementation and execution of the just learned theories. The main concern here was, which software to use. I therefore decided to reread the empirical studies and to identify the software the researchers used. One software platform that was used several times was the G@ARCH 7 module of the OxMetrics software package. After testing the demo version of this software I decided that this would be the best way to conduct my own research. A particular advantage of this software was the focus on the estimation and testing of GARCH-type volatility models. Together with the Microsoft Excel it was then possible to start my research.

After conducting more than 60 estimations and tests during my research, I am now able to use the software package in a proficient way. It can thus be said that I developed further skills in the application of OxMetrics and Microsoft Excel, with which I could familiarize myself already during my Masters course at DBS.

Other problems that occurred to me during the course of writing were tackled in a similar way, i.e. analyzing previous studies with regard to their methodology. In cases where this did not lead to sufficient solutions my supervisor and mentor provided me with thought-provoking and creative impulses. So it was possible to come to valuable conclusions about methodology in an active dialog.

Summary of Added Value

As stated earlier does literature show no consensus on which value at risk approach can be championed. Especially, the incorporation of extreme market conditions, i.e. the financial crisis of 2008, was not discussed excessively to the knowledge of the author. As a consequence I was not really able to form any expectations regarding the outcome of my own research. My findings thus contribute to a specialized problem of a larger research field and confirmed already existing opinions about the role of asymmetry in volatility dynamics.

Conclusion

Due to the fact that I had only superficial knowledge about my chosen topic, it is possible for me to say that I gained a lot of knowledge and research skills by conducting my thesis. By acquiring the knowledge about value at risk and all its components on my own and then was practically applied, the know-how was gained in a sustainable way. However, the research on my thesis also showed me that there are many things I still have to learn, which can enable me a greater understanding of financial markets.

Appendix B.I Oxmetrics Output Crisis Sample

Ox Professional version 7.00 (Windows_64/U/MT) (C) J.A. Doornik, 1994-2013
Copyright for this package: S. Laurent, 2000-2012.
G@RCH package version 7.0, object created on 31-07-2014
Copyright for this package: S. Laurent, 2000-2012.

Starting estimation process...

```
*****  
** G@RCH(1) SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives

Log-likelihood = 4460.49

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000833	0.00030200	2.758	0.0059
Cst(V) x 10 ⁴	0.023974	0.010232	2.343	0.0192
ARCH(Alpha1)	0.085924	0.017055	5.038	0.0000
GARCH(Beta1)	0.903606	0.017523	51.57	0.0000

No. Observations :	1531	No. Parameters :	4
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4460.486	Alpha[1]+Beta[1]:	0.98953

The sample mean of squared residuals was used to start recursion.

The positivity constraint for the GARCH (1,1) is observed.

This constraint is $\alpha[L]/[1 - \beta(L)] \geq 0$.

The unconditional variance is 0.00022899

The conditions are $\alpha[0] > 0$, $\alpha[L] + \beta[L] < 1$ and $\alpha[i] + \beta[i] \geq 0$.

=> See Doornik & Ooms (2001) for more details.

The condition for existence of the fourth moment of the GARCH is observed.

The constraint equals 0.993936 and should be < 1 .

=> See Ling & McAleer (2001) for details.

Estimated Parameters Vector :

0.000833; 0.023974; 0.085924; 0.903611

Elapsed Time : 0.11 seconds (or 0.00183333 minutes).

CondV [1 - 1531] saved to Returns with Crisis.xls

VaR_in(0.05) [1 - 1531] saved to Returns with Crisis.xls

Starting estimation process...

** GARCH(2) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 6.92689 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 4482.15
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000938	0.00027929	3.357	0.0008
Cst(V) x 10 ⁴	0.017486	0.0094276	1.855	0.0638
ARCH(Alpha1)	0.086543	0.016734	5.172	0.0000
GARCH(Beta1)	0.909117	0.015746	57.74	0.0000
Student(DF)	6.926894	1.1982	5.781	0.0000

No. Observations :	1531	No. Parameters :	5
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4482.152	Alpha[1]+Beta[1]:	0.99566

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is $\alpha[L]/[1 - \beta(L)] \geq 0$.
The unconditional variance is 0.000402882
The conditions are $\alpha[0] > 0$, $\alpha[L] + \beta[L] < 1$ and $\alpha[i] + \beta[i] \geq 0$.

=> See Doornik & Ooms (2001) for more details.

The condition for existence of the fourth moment of the GARCH is not observed.

The constraint equals 1.02167 and should be < 1.

=> See Ling & McAleer (2001) for details.

Estimated Parameters Vector :

0.000938; 0.017486; 0.086543; 0.909117; 6.926899

Elapsed Time : 0.148 seconds (or 0.00246667 minutes).

Starting estimation process...

** GARCH(3) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 7.30777 degrees of freedom.
and asymmetry coefficient (log xi) -0.0737584.

Strong convergence using numerical derivatives
Log-likelihood = 4484.58
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.000750	0.00030134	2.489	0.0129
Cst (V) x 10 ⁴	0.016929	0.0090065	1.880	0.0604
ARCH (Alpha)	0.083684	0.016147	5.183	0.0000
GARCH (Beta)	0.910948	0.015432	59.03	0.0000
Asymmetry	-0.073758	0.030221	-2.441	0.0148
Tail	7.307771	1.3228	5.525	0.0000

No. Observations :	1531	No. Parameters :	6
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4484.578	Alpha[1]+Beta[1]:	0.99463

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is $\alpha[L]/[1 - \beta(L)] \geq 0$.
The unconditional variance is 0.000315328
The conditions are $\alpha[0] > 0$, $\alpha[L] + \beta[L] < 1$ and $\alpha[i] + \beta[i] \geq 0$.

=> See Doornik & Ooms (2001) for more details.

The condition for existence of the fourth moment of the GARCH is not observed.

The constraint equals 1.01637 and should be < 1.

=> See Ling & McAleer (2001) for details.

Estimated Parameters Vector :

0.000750; 0.016929; 0.083684; 0.910948;-0.073758; 7.307776

Elapsed Time : 0.292 seconds (or 0.00486667 minutes).

Starting estimation process...

** GARCH(4) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: RiskMetrics (lambda=0.94).
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4448.11
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.000675	0.00038375	1.760	0.0786
ARCH(Alpha)	0.060000			
GARCH(Beta)	0.940000			

No. Observations :	1531	No. Parameters :	1
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4448.109		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
0.000680

Elapsed Time : 0.026 seconds (or 0.000433333 minutes).

Starting estimation process...

```
*****
** GARCH(5) SPECIFICATIONS **
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: IGARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4448.42
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000681	0.00039789	1.710	0.0874
ARCH(Alpha1)	0.065572	0.011776	5.568	0.0000
GARCH(Beta1)	0.934428			

No. Observations :	1531	No. Parameters :	2
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4448.424		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :

0.000681; 0.065577

Elapsed Time : 0.045 seconds (or 0.00075 minutes).

Starting estimation process...

 ** GARCH(7) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
 Thesis\Data\Price and Return Data>Returns with Crisis.xls
 The estimation sample is: 2008-01-02 - 2013-12-31
 The dependent variable is: CDAX
 Mean Equation: ARMA (0, 0) model.
 No regressor in the conditional mean
 Variance Equation: IGARCH (1, 1) model.
 No regressor in the conditional variance
 Skewed Student distribution, with 7.69717 degrees of freedom.
 and asymmetry coefficient (log xi) -0.0754478.

Strong convergence using numerical derivatives
 Log-likelihood = 4479.46
 Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000786	0.00032254	2.436	0.0149
ARCH(Alpha1)	0.069553	0.011885	5.852	0.0000
Asymmetry	-0.075448	0.029203	-2.584	0.0099
Tail	7.697171	1.2209	6.304	0.0000
GARCH(Beta1)	0.930447			

No. Observations : 1531 No. Parameters : 4
 Mean (Y) : 0.00011 Variance (Y) : 0.00025
 Skewness (Y) : 0.00457 Kurtosis (Y) : 8.13024
 Log Likelihood : 4479.463

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
 0.000786; 0.069553;-0.075448; 7.697176
 Elapsed Time : 0.139 seconds (or 0.00231667 minutes).

Starting estimation process...

** GARCH(6) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: IGARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 7.41933 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 4476.63
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)				
	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000930	0.00030624	3.037	0.0024
ARCH(Alpha1)	0.069851	0.011963	5.839	0.0000
Student(DF)	7.419326	1.1200	6.624	0.0000
GARCH(Beta1)	0.930149			

No. Observations :	1531	No. Parameters :	3
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4476.628		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
0.000930; 0.069851; 7.419331
Elapsed Time : 0.064 seconds (or 0.00106667 minutes).

Starting estimation process...

** GARCH(9) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4455.69
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000882	0.00034599	2.551	0.0109
d-Figarch	0.521301	0.062099	8.395	0.0000
ARCH(Phil)	-0.003991	0.061661	-0.06473	0.9484
GARCH(Beta1)	0.495513	0.082149	6.032	0.0000

No. Observations : 1531 No. Parameters : 4
Mean (Y) : 0.00011 Variance (Y) : 0.00025
Skewness (Y) : 0.00457 Kurtosis (Y) : 8.13024
Log Likelihood : 4455.686

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
not observed.

=> See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :
0.000882; 0.521301;-0.003991; 0.495518
Elapsed Time : 3.334 seconds (or 0.0555667 minutes).

Starting estimation process...

** G@RCH(11) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.
No regressor in the conditional variance
Skewed Student distribution, with 7.94896 degrees of freedom.
and asymmetry coefficient (log xi) -0.0980242.

Strong convergence using numerical derivatives
Log-likelihood = 4488.28
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.000694	0.00031698	2.188	0.0288
d-Figarch	0.516309	0.060261	8.568	0.0000
ARCH (Phi)	-0.033561	0.057163	-0.5871	0.5572
GARCH (Beta)	0.505051	0.094083	5.368	0.0000
Asymmetry	-0.098024	0.029840	-3.285	0.0010
Tail	7.948957	1.3131	6.053	0.0000

No. Observations :	1531	No. Parameters :	6
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4488.282		

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is not observed.

=> See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :

0.000694; 0.516309;-0.033561; 0.505051;-0.098024; 7.948962

Elapsed Time : 7.402 seconds (or 0.123367 minutes).

Starting estimation process...

```

*****
** GARCH(10) SPECIFICATIONS **
*****
The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.
No regressor in the conditional variance
Student distribution, with 7.74902 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 4484.21
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)
      Coefficient  Std.Error  t-value  t-prob
Cst(M)           0.000920  0.00029174   3.153  0.0016
d-Figarch        0.525021  0.065538    8.011  0.0000
ARCH(Phil)      -0.004227  0.069852  -0.06051  0.9518
GARCH(Beta1)    0.511930  0.10382    4.931  0.0000
Student(DF)     7.749020  1.2584     6.158  0.0000

No. Observations :      1531  No. Parameters   :          5
Mean (Y)          :  0.00011  Variance (Y)    :  0.00025
Skewness (Y)     :  0.00457  Kurtosis (Y)   :  8.13024
Log Likelihood   :  4484.211

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
not observed.
=> See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :
0.000920; 0.525021;-0.004227; 0.511930; 7.749025
Elapsed Time : 3.697 seconds (or 0.0616167 minutes).

```

Starting estimation process...

** GARCH(3) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: EGARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical
derivatives
Log-likelihood = 4469.18
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000099	0.00021463	0.4596	0.6459
Cst(V) x 10 ⁴	-86079.976237	293.39	-293.4	0.0000
ARCH(Alpha1)	0.535838	0.60328	0.8882	0.3746
GARCH(Beta1)	0.947716	0.0098340	96.37	0.0000
EGARCH(Theta1)	-0.103322	0.052993	-1.950	0.0514
EGARCH(Theta2)	0.141355	0.048085	2.940	0.0033

No. Observations : 1531 No. Parameters : 6
Mean (Y) : 0.00011 Variance (Y) : 0.00025
Skewness (Y) : 0.00457 Kurtosis (Y) : 8.13024
Log Likelihood : 4469.183

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
0.000099;-86079.976237; 0.535838; 0.947716;-0.103322; 0.141355
Elapsed Time : 0.587 seconds (or 0.00978333 minutes).
CondV [1 - 1531] saved to Returns with Crisis.xls
VaR_in(0.05) [1 - 1531] saved to Returns with Crisis.xls

Starting estimation process...

 ** GARCH(4) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns with Crisis.xls
 The estimation sample is: 2008-01-02 - 2013-12-31
 The dependent variable is: CDAX
 Mean Equation: ARMA (0, 0) model.
 No regressor in the conditional mean
 Variance Equation: EGARCH (1, 1) model.
 No regressor in the conditional variance
 Skewed Student distribution, with 7.98053 degrees of freedom.
 and asymmetry coefficient (log xi) -0.119674.

Weak convergence (no improvement in line search) using numerical derivatives
 Log-likelihood = 4491.08
 Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000131	0.00024814	0.5268	0.5984
Cst(V) x 10 ⁴	-93593.761258	181.36	-516.1	0.0000
ARCH(Alpha1)	0.686706	0.59323	1.158	0.2472
GARCH(Beta1)	0.944090	0.0098894	95.46	0.0000
EGARCH(Theta1)	-0.119537	0.053223	-2.246	0.0248
EGARCH(Theta2)	0.129308	0.039242	3.295	0.0010
Asymmetry	-0.119674	0.028203	-4.243	0.0000
Tail	7.980527	1.5923	5.012	0.0000

No. Observations : 1531 No. Parameters : 8
 Mean (Y) : 0.00011 Variance (Y) : 0.00025
 Skewness (Y) : 0.00457 Kurtosis (Y) : 8.13024
 Log Likelihood : 4491.078

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
 0.000131;-93593.761258; 0.686706; 0.944090;-0.119537; 0.129308;-0.119674;
 7.980527

Elapsed Time : 1.72 seconds (or 0.0286667 minutes).

Starting estimation process...

```

*****
** GARCH(3) SPECIFICATIONS **
*****
The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: EGARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 2.00027 degrees of freedom.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 4459.02

No. Observations :      1531  No. Parameters :      7
Mean (Y)          :    0.00011  Variance (Y)      :    0.00025
Skewness (Y)     :    0.00457  Kurtosis (Y)     :    8.13024
Log Likelihood   :    4459.019

Estimated Parameters Vector :
  0.000790; 2.202976; 0.066603; 0.979397;-9.885967; 6.777190; 2.000273
Parameters Names
Cst(M)           ; Cst(V) x 10^4      ; ARCH(Alpha1)      ; GARCH(Beta1)
; EGARCH(Theta1) ; EGARCH(Theta2)    ; Student(DF)      ;
The tests are not reported since there is no convergence.
Elapsed Time : 0.546 seconds (or 0.0091 minutes).

```

Starting estimation process...

** GARCH(7) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GJR (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4502.17
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)				
	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000315	0.00027989	1.126	0.2603
Cst(V) x 10 ⁴	0.026848	0.010646	2.522	0.0118
ARCH(Alpha1)	-0.025026	0.0092860	-2.695	0.0071
GARCH(Beta1)	0.925940	0.019731	46.93	0.0000
GJR(Gamma1)	0.165750	0.033216	4.990	0.0000

No. Observations :	1531	No. Parameters :	5
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4502.174		

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.5$ with this distribution.)
In this estimation, this sum equals 0.983788.
The condition for existence of the fourth moment of the GJR is observed.
The constraint equals 0.995137 (should be < 1). => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
0.000315; 0.026848;-0.025026; 0.925940; 0.165755
Elapsed Time : 0.177 seconds (or 0.00295 minutes).
CondV [1 - 1531] saved to Returns with Crisis.xls
VaR_in(0.05) [1 - 1531] saved to Returns with Crisis.xls

Starting estimation process...

** GARCH(9) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GJR (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 9.12334 degrees of freedom.
and asymmetry coefficient (log xi) -0.107163.

Weak convergence (no improvement in line search) using numerical derivatives

Log-likelihood = 4521.74

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000371	0.00028073	1.323	0.1861
Cst(V) x 10 ⁴	0.022243	0.010531	2.112	0.0348
ARCH(Alpha)	-0.029701	0.0088577	-3.353	0.0008
GARCH(Beta)	0.928558	0.021467	43.25	0.0000
GJR(Gamma)	0.177078	0.038725	4.573	0.0000
Asymmetry	-0.107163	0.032369	-3.311	0.0010
Tail	9.123339	1.9448	4.691	0.0000

No. Observations :	1531	No. Parameters :	7
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4521.743		

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.553377$ with this distribution.)

In this estimation, this sum equals 0.996848.

The condition for existence of the fourth moment of the GJR is not observed.

The constraint equals 1.04173 (should be < 1). => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :

0.000371; 0.022243;-0.029701; 0.928558; 0.177078;-0.107163; 9.123339
Elapsed Time : 0.631 seconds (or 0.0105167 minutes).

Starting estimation process...

** GARCH(8) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GJR (1, 1) model.
No regressor in the conditional variance
Student distribution, with 8.17476 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 4517.01
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000596	0.00026581	2.243	0.0250
Cst(V) x 10 ⁴	0.020837	0.010077	2.068	0.0388
ARCH(Alpha1)	-0.029428	0.0093568	-3.145	0.0017
GARCH(Beta1)	0.927913	0.020394	45.50	0.0000
GJR(Gamma1)	0.176588	0.037022	4.770	0.0000
Student(DF)	8.174758	1.5938	5.129	0.0000

No. Observations : 1531 No. Parameters : 6
Mean (Y) : 0.00011 Variance (Y) : 0.00025
Skewness (Y) : 0.00457 Kurtosis (Y) : 8.13024
Log Likelihood : 4517.015

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.5$ with this distribution.)
In this estimation, this sum equals 0.986779.
The condition for existence of the fourth moment of the GJR is not observed.
The constraint equals 1.02023 (should be < 1) while the degree of freedom is 8.17476 (should be ≥ 5) => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
0.000596; 0.020837;-0.029428; 0.927913; 0.176588; 8.174763

Elapsed Time : 0.245 seconds (or 0.00408333 minutes).

Ox Professional version 7.00 (Windows_64/U/MT) (C) J.A. Doornik, 1994-2013
Copyright for this package: S. Laurent, 2000-2012.
G@RCH package version 7.0, object created on 12-08-2014
Copyright for this package: S. Laurent, 2000-2012.

Starting estimation process...

** SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: APARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 4509.18

No. Observations :	1531	No. Parameters :	5
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4509.181		

Estimated Parameters Vector :
0.000128; 3.005805; 0.066488; 0.926872; 1.104250
Parameters Names
Cst(M) ; Cst(V) x 10⁴ ; ARCH(Alphal) ; GARCH(Betal)
; APARCH(Gamma1) ;

The tests are not reported since there is no convergence.
Elapsed Time : 0.16 seconds (or 0.00266667 minutes).
CondV [1 - 1531] saved to Returns with Crisis.xls
VaR_in(0.05) [1 - 1531] saved to Returns with Crisis.xls

Starting estimation process...

** GARCH(3) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns with Crisis.xls
The estimation sample is: 2008-01-02 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: APARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 9.93713 degrees of freedom.
and asymmetry coefficient (log xi) -0.11846.

Strong convergence using numerical derivatives
Log-likelihood = 4527.23
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000193	0.00026599	0.7271	0.4673
Cst(V) x 10 ⁴	2.798175	0.97723	2.863	0.0042
ARCH(Alpha1)	0.067813	0.014705	4.611	0.0000
GARCH(Beta1)	0.927759	0.016312	56.88	0.0000
APARCH(Gamma1)	1.196452	0.16613	7.202	0.0000
Asymmetry	-0.118460	0.032307	-3.667	0.0003
Tail	9.937131	2.2953	4.329	0.0000
APARCH(Delta)	1.000000			

No. Observations :	1531	No. Parameters :	7
Mean (Y) :	0.00011	Variance (Y) :	0.00025
Skewness (Y) :	0.00457	Kurtosis (Y) :	8.13024
Log Likelihood :	4527.230		

The sample mean of squared residuals was used to start recursion.
The condition for existence of $E(\sigma^\Delta)$ and $E(|e^\Delta|)$ is
observed.

The constraint equals 0.996192 and should be < 1.

Estimated Parameters Vector :
0.000193; 2.798175; 0.067813; 0.927759; 1.196452; -0.118460; 9.937136
Elapsed Time : 0.665 seconds (or 0.0110833 minutes).
CondV [1 - 1531] saved to Returns with Crisis.xls
VaR_in(0.05) [1 - 1531] saved to Returns with Crisis.xls

Starting estimation process...

```
*****  
** GARCH(1) SPECIFICATIONS **  
*****  
The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master  
Thesis\Data\Price and Return Data>Returns with Crisis.xls  
The estimation sample is: 2008-01-02 - 2013-12-31  
The dependent variable is: CDAX  
Mean Equation: ARMA (0, 0) model.  
No regressor in the conditional mean  
Variance Equation: APARCH (1, 1) model.  
No regressor in the conditional variance  
Student distribution, with 8.93153 degrees of freedom.
```

Strong convergence using numerical derivatives
Log-likelihood = 4521.45
Please wait : Computing the Std Errors ...

```
Robust Standard Errors (Sandwich formula)  
Coefficient Std.Error t-value t-prob  
Cst(M) 0.000420 0.00025508 1.645 0.1001  
Cst(V) x 10^4 2.643767 0.94375 2.801 0.0052  
ARCH(Alpha1) 0.068887 0.014368 4.795 0.0000  
GARCH(Beta1) 0.926914 0.015826 58.57 0.0000  
APARCH(Gamma1) 1.162065 0.16598 7.001 0.0000  
Student(DF) 8.931526 1.8836 4.742 0.0000  
APARCH(Delta) 1.000000
```

```
No. Observations : 1531 No. Parameters : 6  
Mean (Y) : 0.00011 Variance (Y) : 0.00025  
Skewness (Y) : 0.00457 Kurtosis (Y) : 8.13024  
Log Likelihood : 4521.449
```

The sample mean of squared residuals was used to start recursion.
The condition for existence of $E(\sigma^{\Delta})$ and $E(|e^{\Delta}|)$ is observed.

The constraint equals 0.979937 and should be < 1 .

Estimated Parameters Vector :

0.000420; 2.643767; 0.068887; 0.926914; 1.162065; 8.931531

Elapsed Time : 0.211 seconds (or 0.00351667 minutes).

Appendix B.II Oxmetrics Output Pre-Crisis Sample

Returns since 2003 - Kopie.xls saved to C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns Precrisis.xls
Ox Professional version 7.00 (Windows_64/U/MT) (C) J.A. Doornik, 1994-2013
Copyright for this package: S. Laurent, 2000-2012.
G@RCH package version 7.0, object created on 31-07-2014
Copyright for this package: S. Laurent, 2000-2012.

Starting estimation process...

```
*****  
** G@RCH(1) SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4141.88
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000993	0.00024848	3.997	0.0001
Cst(V) x 10 ⁴	0.021937	0.0079557	2.757	0.0059
ARCH(Alpha1)	0.076189	0.015832	4.812	0.0000
GARCH(Beta1)	0.901464	0.018389	49.02	0.0000

No. Observations :	1292	No. Parameters :	4
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4141.875	Alpha[1]+Beta[1]:	0.97765

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is $\alpha[L]/[1 - \beta(L)] \geq 0$.
The unconditional variance is 9.81633e-005
The conditions are $\alpha[0] > 0$, $\alpha[L] + \beta[L] < 1$ and $\alpha[i] + \beta[i] \geq 0$.

=> See Doornik & Ooms (2001) for more details.

The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.967414 and should be < 1.

=> See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
0.000993; 0.021937; 0.076189; 0.901469
Elapsed Time : 0.079 seconds (or 0.00131667 minutes).
CondV [1 - 1292] saved to Returns Precrisis.xls
VaR_in(0.05) [1 - 1292] saved to Returns Precrisis.xls

Starting estimation process...

** GARCH(2) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 8.63058 degrees of freedom.
and asymmetry coefficient (log xi) -0.140172.

Strong convergence using numerical derivatives
Log-likelihood = 4170.72
Please wait : Computing the Std Errors ...

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000996	0.00024403	4.080	0.0000
Cst(V) x 10 ⁴	0.015523	0.0063870	2.430	0.0152
ARCH(Alpha1)	0.080579	0.015905	5.066	0.0000
GARCH(Beta1)	0.905782	0.017925	50.53	0.0000

Asymmetry	-0.140172	0.035327	-3.968	0.0001
Tail	8.630576	2.3156	3.727	0.0002

No. Observations :	1292	No. Parameters :	6
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4170.715	Alpha[1]+Beta[1]:	0.98636

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is $\alpha[L]/[1 - \beta(L)] \geq 0$.
The unconditional variance is 0.000113814
The conditions are $\alpha[0] > 0$, $\alpha[L] + \beta[L] < 1$ and $\alpha[i] + \beta[i] \geq 0$.

=> See Doornik & Ooms (2001) for more details.

The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.995187 and should be < 1 .

=> See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
0.000996; 0.015523; 0.080579; 0.905782;-0.140172; 8.630581
Elapsed Time : 0.269 seconds (or 0.00448333 minutes).

Ox Professional version 7.00 (Windows_64/U/MT) (C) J.A. Doornik, 1994-2013
Copyright for this package: S. Laurent, 2000-2012.
G@RCH package version 7.0, object created on 31-07-2014
Copyright for this package: S. Laurent, 2000-2012.

Starting estimation process...

```
*****
** G@RCH(1) SPECIFICATIONS **
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 7.74851 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 4163.81
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)				
	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001227	0.00023408	5.240	0.0000
Cst(V) x 10^4	0.016671	0.0068990	2.416	0.0158
ARCH(Alpha1)	0.082860	0.016791	4.935	0.0000

GARCH(Beta1)	0.903618	0.018624	48.52	0.0000
Student(DF)	7.748511	1.7992	4.307	0.0000

No. Observations :	1292	No. Parameters :	5
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4163.812	Alpha[1]+Beta[1]:	0.98648

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is $\alpha[L]/[1 - \beta(L)] \geq 0$.
The unconditional variance is 0.000123292
The conditions are $\alpha[0] > 0$, $\alpha[L] + \beta[L] < 1$ and $\alpha[i] + \beta[i] \geq 0$.

=> See Doornik & Ooms (2001) for more details.

The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.997862 and should be < 1 .

=> See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
0.001227; 0.016671; 0.082860; 0.903618; 7.748516
Elapsed Time : 0.112 seconds (or 0.00186667 minutes).
CondV [1 - 1292] saved to Returns Precrisis.xls
VaR_in(0.05) [1 - 1292] saved to Returns Precrisis.xls

Starting estimation process...

** GARCH(3) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: RiskMetrics (lambda=0.94).
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4126.24
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000835	0.00033563	2.486	0.0130
ARCH(Alpha)	0.060000			
GARCH(Beta)	0.940000			

No. Observations :	1292	No. Parameters :	1
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4126.244		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :

0.000840

Elapsed Time : 0.023 seconds (or 0.000383333 minutes).

Starting estimation process...

** GARCH(4) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: IGARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical
derivatives

Log-likelihood = 4126.24

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.000839	0.00033191	2.529	0.0115
ARCH(Alphal)	0.060070	0.014988	4.008	0.0001
GARCH(Betal)	0.939930			

No. Observations :	1292	No. Parameters :	2
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4126.244		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :

0.000839; 0.060070

Elapsed Time : 0.091 seconds (or 0.00151667 minutes).

Starting estimation process...

** GARCH(6) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: IGARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 8.59273 degrees of freedom.
and asymmetry coefficient (log xi) -0.137751.

Strong convergence using numerical derivatives
Log-likelihood = 4164.33
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001034	0.00026710	3.871	0.0001
ARCH(Alpha1)	0.065987	0.013067	5.050	0.0000
Asymmetry	-0.137751	0.033364	-4.129	0.0000
Tail	8.592727	1.8771	4.578	0.0000
GARCH(Beta1)	0.934013			

No. Observations :	1292	No. Parameters :	4
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4164.327		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
0.001034; 0.065987;-0.137751; 8.592732
Elapsed Time : 0.154 seconds (or 0.00256667 minutes).

Starting estimation process...

** GARCH(5) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: IGARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 7.8758 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 4156.83
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001207	0.00026306	4.587	0.0000
ARCH(Alpha1)	0.065333	0.013300	4.912	0.0000
Student(DF)	7.875795	1.5130	5.205	0.0000
GARCH(Beta1)	0.934667			

No. Observations :	1292	No. Parameters :	3
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4156.835		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
0.001207; 0.065333; 7.875800
Elapsed Time : 0.061 seconds (or 0.00101667 minutes).

Starting estimation process...

** GARCH(7) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls

The estimation sample is: 2003-01-01 - 2007-12-31

The dependent variable is: CDAX

Mean Equation: ARMA (0, 0) model.

No regressor in the conditional mean

Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.

No regressor in the conditional variance

Normal distribution.

Strong convergence using numerical derivatives

Log-likelihood = 4129.26

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000911	0.00029654	3.073	0.0022
d-Figarch	0.539632	0.12325	4.378	0.0000
ARCH(Phil)	0.241769	0.073690	3.281	0.0011
GARCH(Beta1)	0.716568	0.089016	8.050	0.0000

No. Observations :	1292	No. Parameters :	4
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4129.261		

The sample mean of squared residuals was used to start recursion.

The positivity constraint for the FIGARCH (1,d,1) is not observed.

=> See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :

0.000911; 0.539632; 0.241769; 0.716573

Elapsed Time : 2.551 seconds (or 0.0425167 minutes).

Starting estimation process...

** GARCH(9) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls

The estimation sample is: 2003-01-01 - 2007-12-31

The dependent variable is: CDAX

Mean Equation: ARMA (0, 0) model.

No regressor in the conditional mean

Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.

No regressor in the conditional variance

Skewed Student distribution, with 8.74231 degrees of freedom.

and asymmetry coefficient (log xi) -0.138893.

Strong convergence using numerical derivatives

Log-likelihood = 4167.95

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001040	0.00025530	4.074	0.0000
d-Figarch	0.523535	0.077698	6.738	0.0000
ARCH(Phil)	0.185084	0.055229	3.351	0.0008
GARCH(Betal)	0.658106	0.075735	8.690	0.0000
Asymmetry	-0.138893	0.034020	-4.083	0.0000
Tail	8.742312	2.0511	4.262	0.0000

No. Observations :	1292	No. Parameters :	6
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4167.951		

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
not observed.

=> See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :

0.001040; 0.523535; 0.185084; 0.658106;-0.138893; 8.742317

Elapsed Time : 4.548 seconds (or 0.0758 minutes).

Starting estimation process...

** GARCH(8) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls

The estimation sample is: 2003-01-01 - 2007-12-31

The dependent variable is: CDAX

Mean Equation: ARMA (0, 0) model.

No regressor in the conditional mean

Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.

No regressor in the conditional variance

Student distribution, with 8.04231 degrees of freedom.

Strong convergence using numerical derivatives

Log-likelihood = 4160.51

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001223	0.00025362	4.821	0.0000
d-Figarch	0.514224	0.075283	6.830	0.0000
ARCH(Phil)	0.179604	0.058006	3.096	0.0020
GARCH(Beta1)	0.644568	0.080336	8.023	0.0000
Student(DF)	8.042310	1.6056	5.009	0.0000

No. Observations :	1292	No. Parameters :	5
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4160.509		

The sample mean of squared residuals was used to start recursion.

The positivity constraint for the FIGARCH (1,d,1) is not observed.

=> See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :

0.001223; 0.514224; 0.179604; 0.644568; 8.042315

Elapsed Time : 2.691 seconds (or 0.04485 minutes).

Starting estimation process...

** SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: EGARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 4078.98

No. Observations :	1292	No. Parameters :	6
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4078.982		

Estimated Parameters Vector :

0.000766;-0.008365; 0.230844; 0.997820;-0.034666; 0.285186

Parameters Names

Cst(M) ; Cst(V) x 10⁴ ; ARCH(Alpha1) ; GARCH(Beta1)
; EGARCH(Theta1) ; EGARCH(Theta2) ;

The tests are not reported since there is no convergence.

Elapsed Time : 0.163 seconds (or 0.00271667 minutes).

CondV [1 - 1292] saved to Returns Precrisis.xls

VaR_in(0.05) [1 - 1292] saved to Returns Precrisis.xls

Starting estimation process...

** GARCH(2) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: EGARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 11.078 degrees of freedom.
and asymmetry coefficient (log xi) -0.156815.

Weak convergence (no improvement in line search) using numerical derivatives

Log-likelihood = 4157.29

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000736	0.00023351	3.153	0.0017
Cst(V) x 10 ⁴	-100902.952944	115.30	-875.1	0.0000
ARCH(Alpha1)	0.237656	0.33471	0.7100	0.4778
GARCH(Beta1)	0.940124	0.014044	66.94	0.0000
EGARCH(Theta1)	-0.109315	0.046790	-2.336	0.0196
EGARCH(Theta2)	0.178913	0.041179	4.345	0.0000
Asymmetry	-0.156815	0.030606	-5.124	0.0000
Tail	11.078046	3.7791	2.931	0.0034

No. Observations :	1292	No. Parameters :	8
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4157.289		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :

0.000736;-100902.952944; 0.237656; 0.940124;-0.109315; 0.178913;-0.156815; 11.078046

Elapsed Time : 1.431 seconds (or 0.02385 minutes).

Starting estimation process...

** GARCH(1) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: EGARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 9.00295 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 4148.84
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000992	0.00021831	4.543	0.0000
Cst(V) x 10^4	-93454.262808	635.70	-147.0	0.0000
ARCH(Alpha1)	0.285045	0.38156	0.7471	0.4552
GARCH(Beta1)	0.936747	0.019022	49.25	0.0000
EGARCH(Theta1)	-0.098095	0.047953	-2.046	0.0410
EGARCH(Theta2)	0.180661	0.045619	3.960	0.0001
Student(DF)	9.002947	2.6684	3.374	0.0008

No. Observations :	1292	No. Parameters :	7
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4148.836		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :

0.000992;-93454.262808; 0.285045; 0.936747;-0.098095; 0.180661; 9.002952
Elapsed Time : 1.072 seconds (or 0.0178667 minutes).

Starting estimation process...

** GARCH(4) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GJR (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4153.39
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.000738	0.00024700	2.988	0.0029
Cst (V) x 10 ⁴	0.023814	0.0083236	2.861	0.0043
ARCH (Alpha1)	0.018023	0.012155	1.483	0.1384
GARCH (Beta1)	0.902832	0.017525	51.52	0.0000
GJR (Gamma1)	0.104820	0.029756	3.523	0.0004

No. Observations :	1292	No. Parameters :	5
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4153.387		

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.5$ with this distribution.)
In this estimation, this sum equals 0.973265.
The condition for existence of the fourth moment of the GJR is observed.
The constraint equals 0.965407 (should be < 1). => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :

0.000738; 0.023814; 0.018023; 0.902832; 0.104825

Elapsed Time : 0.145 seconds (or 0.00241667 minutes).

CondV [1 - 1292] saved to Returns Precrisis.xls

VaR_in(0.05) [1 - 1292] saved to Returns Precrisis.xls

Starting estimation process...

** GARCH(6) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns Precrisis.xls

The estimation sample is: 2003-01-01 - 2007-12-31

The dependent variable is: CDAX

Mean Equation: ARMA (0, 0) model.

No regressor in the conditional mean

Variance Equation: GJR (1, 1) model.

No regressor in the conditional variance

Skewed Student distribution, with 9.48664 degrees of freedom.
and asymmetry coefficient (log xi) -0.162563.

Strong convergence using numerical derivatives

Log-likelihood = 4182.03

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.000746	0.00024577	3.033	0.0025
Cst (V) x 10 ⁴	0.018390	0.0064660	2.844	0.0045
ARCH (Alpha)	0.013543	0.011507	1.177	0.2394
GARCH (Beta)	0.907346	0.016902	53.68	0.0000
GJR (Gamma)	0.121082	0.030402	3.983	0.0001
Asymmetry	-0.162563	0.036202	-4.490	0.0000
Tail	9.486641	2.8414	3.339	0.0009

No. Observations :	1292	No. Parameters :	7
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4182.031		

The sample mean of squared residuals was used to start recursion.

The condition for existence of the second moment of the GJR is observed.
This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.580573$ with this distribution.)

In this estimation, this sum equals 0.991186.

The condition for existence of the fourth moment of the GJR is not observed.

The constraint equals 1.02046 (should be < 1). => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :

0.000746; 0.018390; 0.013543; 0.907346; 0.121082; -0.162563; 9.486646

Elapsed Time : 0.323 seconds (or 0.00538333 minutes).

Starting estimation process...

** GARCH(5) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GJR (1, 1) model.
No regressor in the conditional variance
Student distribution, with 8.59295 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 4172.98
Please wait : Computing the Std Errors ...

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001006	0.00024125	4.169	0.0000
Cst(V) x 10 ⁴	0.018663	0.0069291	2.693	0.0072
ARCH(Alpha1)	0.017827	0.013019	1.369	0.1711
GARCH(Beta1)	0.904880	0.018022	50.21	0.0000
GJR(Gamma1)	0.113303	0.031283	3.622	0.0003
Student(DF)	8.592955	2.2519	3.816	0.0001

No. Observations :	1292	No. Parameters :	6
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4172.984		

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.5$ with this distribution.)
In this estimation, this sum equals 0.979357.
The condition for existence of the fourth moment of the GJR is observed.
The constraint equals 0.991302 (should be < 1) while the degree of freedom is 8.59295 (should be ≥ 5) => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
0.001006; 0.018663; 0.017827; 0.904880; 0.113303; 8.592960
Elapsed Time : 0.187 seconds (or 0.00311667 minutes).

Ox Professional version 7.00 (Windows_64/U/MT) (C) J.A. Doornik, 1994-2013
Copyright for this package: S. Laurent, 2000-2012.
G@RCH package version 7.0, object created on 14-08-2014
Copyright for this package: S. Laurent, 2000-2012.

Starting estimation process...

** G@RCH(1) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: APARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical
derivatives
Log-likelihood = 4134.51
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001064	0.00041877	2.541	0.0112
Cst(V) x 10^4	2.543841	0.77741	3.272	0.0011
ARCH(Alpha1)	0.075649	0.016011	4.725	0.0000
GARCH(Beta1)	0.914816	0.015606	58.62	0.0000
APARCH(Gamma1)	0.663016	0.20274	3.270	0.0011
APARCH(Delta)	1.000000			

No. Observations :	1292	No. Parameters :	5
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4134.507		

The sample mean of squared residuals was used to start recursion.
The condition for existence of $E(\sigma^{\delta})$ and $E(|e^{\delta}|)$ is
observed.

The constraint equals 0.975175 and should be < 1.

Estimated Parameters Vector :

0.001064; 2.543841; 0.075649; 0.914816; 0.663016

Elapsed Time : 0.162 seconds (or 0.0027 minutes).

CondV [1 - 1292] saved to Returns Precrisis.xls

VaR_in(0.05) [1 - 1292] saved to Returns Precrisis.xls

Starting estimation process...

```
*****  
** GARCH(1) SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls

The estimation sample is: 2003-01-01 - 2007-12-31

The dependent variable is: CDAX

Mean Equation: ARMA (0, 0) model.

No regressor in the conditional mean

Variance Equation: APARCH (1, 1) model.

No regressor in the conditional variance

Skewed Student distribution, with 8.29741 degrees of freedom.

and asymmetry coefficient (log xi) -0.16069.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 4178.73

No. Observations :	1292	No. Parameters :	7
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4178.728		

Estimated Parameters Vector :

0.000786; 2.177708; 0.079096; 0.918569; 0.765032; -0.160690; 8.297411

Parameters Names

Cst(M) ; Cst(V) x 10⁴ ; ARCH(Alpha1) ; GARCH(Beta1)

; APARCH(Gamma1) ; Asymmetry ; Tail ;

The tests are not reported since there is no convergence.

Elapsed Time : 0.548 seconds (or 0.00913333 minutes).

Starting estimation process...

```
*****  
** GARCH(1) SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns Precrisis.xls
The estimation sample is: 2003-01-01 - 2007-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: APARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 7.70385 degrees of freedom.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 4169.75

No. Observations :	1292	No. Parameters :	6
Mean (Y) :	0.00081	Variance (Y) :	0.00013
Skewness (Y) :	-0.10816	Kurtosis (Y) :	6.36758
Log Likelihood :	4169.754		

Estimated Parameters Vector :

0.001036; 2.148406; 0.080543; 0.916510; 0.714484; 7.703849

Parameters Names

Cst(M) ; Cst(V) x 10⁴ ; ARCH(Alpha) ; GARCH(Beta1)
; APARCH(Gamma1) ; Student(DF) ;

The tests are not reported since there is no convergence.

Elapsed Time : 0.22 seconds (or 0.00366667 minutes).

Appendix B.III Oxmetrics Output Full Sample

Starting estimation process...

```
*****
** GARCH(9) SPECIFICATIONS **
*****
The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.
```

Strong convergence using numerical derivatives
Log-likelihood = 8579.02
Please wait : Computing the Std Errors ...

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000794	0.00024432	3.250	0.0012
ARCH(Alpha1)	0.080433	0.013027	6.175	0.0000
GARCH(Beta1)	0.926812	0.010633	87.16	0.0000

No. Observations :	2823	No. Parameters :	3
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8579.018	Alpha[1]+Beta[1]:	1.00725

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is $\alpha[L]/[1 - \beta(L)] \geq 0$.
The unconditional variance does not exist and/or is not positive.
The conditions are $\alpha[0] > 0$, $\alpha[L] + \beta[L] < 1$ and $\alpha[i] + \beta[i] \geq 0$.

=> See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is not observed.
The constraint equals 1.02748 and should be < 1 .
=> See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
0.000794; 0.080433; 0.926817
Elapsed Time : 0.138 seconds (or 0.0023 minutes).
CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls
VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

** GARCH(10) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 6.69269 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 8636.37
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.001101	0.00019000	5.796	0.0000
ARCH (Alpha)	0.084647	0.012462	6.792	0.0000
GARCH (Beta)	0.924091	0.0098669	93.66	0.0000
Student (DF)	6.692688	0.88654	7.549	0.0000

No. Observations :	2823	No. Parameters :	4
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8636.367	Alpha[1]+Beta[1]:	1.00874

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is $\alpha[L]/[1 - \beta(L)] \geq 0$.
The unconditional variance does not exist and/or is not positive.
The conditions are $\alpha[0] > 0$, $\alpha[L] + \beta[L] < 1$ and $\alpha[i] + \beta[i] \geq 0$.

=> See Doornik & Ooms (2001) for more details.

The condition for existence of the fourth moment of the GARCH is not observed.

The constraint equals 1.04785 and should be < 1 .

=> See Ling & McAleer (2001) for details.

Estimated Parameters Vector :

0.001101; 0.084647; 0.924091; 6.692693

Elapsed Time : 0.199 seconds (or 0.00331667 minutes).

CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls

VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

```
*****  
** GARCH(11) SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: RiskMetrics (lambda=0.94).
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 8572.63
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000773	0.00025489	3.034	0.0024
ARCH(Alpha1)	0.060000			
GARCH(Beta1)	0.940000			

No. Observations :	2823	No. Parameters :	1
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8572.634		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :

0.000778

Elapsed Time : 0.054 seconds (or 0.0009 minutes).

CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls

VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

```
*****  
** GARCH(6) SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: IGARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical derivatives

Log-likelihood = 8596.86

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000914	0.00019698	4.638	0.0000
Cst(V) x 10 ⁴	0.013963	0.0046392	3.010	0.0026
ARCH(Alpha1)	0.097235	0.013702	7.096	0.0000
GARCH(Beta1)	0.902765			

No. Observations :	2823	No. Parameters :	3
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8596.857		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :

0.000914; 0.013963; 0.097235

Elapsed Time : 0.289 seconds (or 0.00481667 minutes).

CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls

VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

** GARCH(12) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: IGARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 7.32142 degrees of freedom.
and asymmetry coefficient (log xi) -0.104849.

Strong convergence using numerical derivatives
Log-likelihood = 8652.16
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000879	0.00019102	4.602	0.0000
Cst(V) x 10 ⁴	0.010529	0.0035035	3.005	0.0027
ARCH(Alpha1)	0.088633	0.011562	7.666	0.0000
Asymmetry	-0.104849	0.023350	-4.490	0.0000
Tail	7.321420	1.0690	6.849	0.0000
GARCH(Beta1)	0.911367			

No. Observations :	2823	No. Parameters :	5
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8652.164		

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
 0.000879; 0.010529; 0.088633;-0.104849; 7.321425
 Elapsed Time : 0.316 seconds (or 0.00526667 minutes).
 CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls
 VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

 ** GARCH(3) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
 Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
 The estimation sample is: 2003-01-01 - 2013-12-31
 The dependent variable is: CDAX
 Mean Equation: ARMA (0, 0) model.
 No regressor in the conditional mean
 Variance Equation: IGARCH (1, 1) model.
 No regressor in the conditional variance
 Student distribution, with 6.84294 degrees of freedom.

Strong convergence using numerical derivatives
 Log-likelihood = 8643.65
 Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001107	0.00017797	6.221	0.0000
Cst(V) x 10^4	0.011896	0.0038689	3.075	0.0021
ARCH(Alpha1)	0.091115	0.011922	7.642	0.0000
Student(DF)	6.842941	0.92701	7.382	0.0000
GARCH(Beta1)	0.908885			

No. Observations : 2823 No. Parameters : 4
 Mean (Y) : 0.00043 Variance (Y) : 0.00019
 Skewness (Y) : -0.04651 Kurtosis (Y) : 8.56106
 Log Likelihood : 8643.655

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :

0.001107; 0.011896; 0.091115; 6.842946

Elapsed Time : 0.197 seconds (or 0.00328333 minutes).

CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls

VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

** GARCH(7) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls

The estimation sample is: 2003-01-01 - 2013-12-31

The dependent variable is: CDAX

Mean Equation: ARMA (0, 0) model.

No regressor in the conditional mean

Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.

No regressor in the conditional variance

Normal distribution.

Strong convergence using numerical derivatives

Log-likelihood = 8609.93

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000907	0.00019527	4.644	0.0000
Cst(V) x 10 ⁴	4.445685	3.0563	1.455	0.1459
d-Figarch	0.554897	0.076580	7.246	0.0000


```
ARCH(Phil)          0.019102   0.063127   0.3026   0.7622
GARCH(Beta1)       0.535558   0.088916   6.023   0.0000
```

```
No. Observations :      2823  No. Parameters :      5
Mean (Y)          :   0.00043  Variance (Y)       :   0.00019
Skewness (Y)     :  -0.04651  Kurtosis (Y)      :   8.56106
Log Likelihood   :  8609.933
```

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is observed.

=> See Chung (1999), Appendix A, for more details.

```
Estimated Parameters Vector :
0.000907; 4.445685; 0.554897; 0.019102; 0.535563
Elapsed Time : 14.351 seconds (or 0.239183 minutes).
CondV [ 1 - 2823] saved to Returns since 2003 - Kopie.xls
VaR_in(0.05) [ 1 - 2823] saved to Returns since 2003 - Kopie.xls
```

Starting estimation process...

```
*****
** G@RCH(11) SPECIFICATIONS **
*****
The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.
No regressor in the conditional variance
Skewed Student distribution, with 7.67649 degrees of freedom.
and asymmetry coefficient (log xi) -0.109876.
```

```
Strong convergence using numerical derivatives
Log-likelihood = 8662.06
Please wait : Computing the Std Errors ...
```

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000865	0.00019284	4.485	0.0000
Cst(V) x 10 ⁴	3.978626	3.2303	1.232	0.2182
d-Figararch	0.575258	0.085052	6.764	0.0000
ARCH(Phil)	0.024200	0.054668	0.4427	0.6580
GARCH(Betal)	0.577510	0.092567	6.239	0.0000
Asymmetry	-0.109876	0.023492	-4.677	0.0000
Tail	7.676489	1.1596	6.620	0.0000

No. Observations : 2823 No. Parameters : 7
Mean (Y) : 0.00043 Variance (Y) : 0.00019
Skewness (Y) : -0.04651 Kurtosis (Y) : 8.56106
Log Likelihood : 8662.059

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is not observed.

=> See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :
0.000865; 3.978626; 0.575258; 0.024200; 0.577510;-0.109876; 7.676494
Elapsed Time : 25.454 seconds (or 0.424233 minutes).
CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls
VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

```
*****
** G@RCH(8) SPECIFICATIONS **
*****
The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: FIGARCH (1, d, 1) model estimated with Chung's method.
No regressor in the conditional variance
Student distribution, with 7.16778 degrees of freedom.
```

Strong convergence using numerical derivatives
Log-likelihood = 8652.74

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.001096	0.00017961	6.104	0.0000
Cst(V) x 10 ⁴	4.212705	3.1419	1.341	0.1801
d-Figarch	0.568926	0.081796	6.955	0.0000
ARCH(Phil)	0.021240	0.057209	0.3713	0.7105
GARCH(Betal)	0.563403	0.097330	5.789	0.0000
Student(DF)	7.167778	1.0004	7.165	0.0000

No. Observations :	2823	No. Parameters :	6
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8652.736		

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is observed.

=> See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :

0.001096; 4.212705; 0.568926; 0.021240; 0.563403; 7.167783

Elapsed Time : 15.949 seconds (or 0.265817 minutes).

CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls

VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Ox Professional version 7.00 (Windows_64/U/MT) (C) J.A. Doornik, 1994-2013
Copyright for this package: S. Laurent, 2000-2012.
G@RCH package version 7.0, object created on 1-08-2014
Copyright for this package: S. Laurent, 2000-2012.

Starting estimation process...

```
*****  
** SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls

The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: EGARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 8541.7

No. Observations :	2823	No. Parameters :	6
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8541.701		

Estimated Parameters Vector :
0.000567;-0.002786; 0.287753; 0.998525;-0.055601; 0.203553
Parameters Names
Cst(M) ; Cst(V) x 10^4 ; ARCH(Alpha1) ; GARCH(Beta1)
; EGARCH(Theta1) ; EGARCH(Theta2) ;
The tests are not reported since there is no convergence.
Elapsed Time : 0.377 seconds (or 0.00628333 minutes).
CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls
VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

```
*****  
** SPECIFICATIONS **  
*****  
The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master  
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls  
The estimation sample is: 2003-01-01 - 2013-12-31  
The dependent variable is: CDAX  
Mean Equation: ARMA (0, 0) model.
```

No regressor in the conditional mean
Variance Equation: EGARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 6.04601 degrees of freedom.
and asymmetry coefficient (log xi) -0.00950226.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 8597.25

No. Observations :	2823	No. Parameters :	8
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8597.247		

Estimated Parameters Vector :
0.001018; 0.000500; 0.324255; 1.000538;-0.077773; 0.213915;-0.009502;
6.046008

Parameters Names

Cst(M) ; Cst(V) x 10⁴ ; ARCH(Alpha1) ; GARCH(Beta1)
; EGARCH(Theta1) ; EGARCH(Theta2) ; Asymmetry ; Tail
;

The tests are not reported since there is no convergence.
Elapsed Time : 1.158 seconds (or 0.0193 minutes).

Ox Professional version 7.00 (Windows_64/U/MT) (C) J.A. Doornik, 1994-2013
Copyright for this package: S. Laurent, 2000-2012.
G@RCH package version 7.0, object created on 1-08-2014
Copyright for this package: S. Laurent, 2000-2012.

Starting estimation process...

** SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls

The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: EGARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 6.03571 degrees of freedom.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 8597.16

No. Observations :	2823	No. Parameters :	7
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8597.161		

Estimated Parameters Vector :
0.001034;-0.007533; 0.320243; 1.000095;-0.078429; 0.213923; 6.035713

Parameters Names

Cst(M) ; Cst(V) x 10⁴ ; ARCH(Alpha1) ; GARCH(Beta1)
; EGARCH(Theta1) ; EGARCH(Theta2) ; Student(DF) ;

The tests are not reported since there is no convergence.

Elapsed Time : 0.739 seconds (or 0.0123167 minutes).

CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls

VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

---- OxMetrics 7.00 started at 16:41:02 on 10-Aug-2014 ----

Ox Professional version 7.00 (Windows_64/U/MT) (C) J.A. Doornik, 1994-2013
Copyright for this package: S. Laurent, 2000-2012.
G@RCH package version 7.0, object created on 10-08-2014
Copyright for this package: S. Laurent, 2000-2012.

Starting estimation process...

** G@RCH(1) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GJR (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical
derivatives
Log-likelihood = 8654.33
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000514	0.00018456	2.786	0.0054
Cst(V) x 10 ⁴	0.024032	0.0064635	3.718	0.0002
ARCH(Alphal)	-0.005249	0.0075574	-0.6946	0.4874
GARCH(Betal)	0.913027	0.013358	68.35	0.0000
GJR(Gammap)	0.146818	0.023904	6.142	0.0000

No. Observations :	2823	No. Parameters :	5
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8654.327		

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.5$ with
this distribution.)
In this estimation, this sum equals 0.981186.
The condition for existence of the fourth moment of the GJR is observed.
The constraint equals 0.988184 (should be < 1). => See Ling & McAleer
(2001) for details.

Estimated Parameters Vector :

0.000514; 0.024032; -0.005249; 0.913027; 0.146818

Elapsed Time : 0.881 seconds (or 0.0146833 minutes).

CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls

VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

 ** GARCH(3) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
 The estimation sample is: 2003-01-01 - 2013-12-31
 The dependent variable is: CDAX
 Mean Equation: ARMA (0, 0) model.
 No regressor in the conditional mean
 Variance Equation: GJR (1, 1) model.
 No regressor in the conditional variance
 Skewed Student distribution, with 9.47707 degrees of freedom.
 and asymmetry coefficient (log xi) -0.135466.

Weak convergence (no improvement in line search) using numerical derivatives

Log-likelihood = 8700.88

Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.000547	0.00018381	2.976	0.0029
Cst (V) x 10 ⁴	0.018967	0.0053040	3.576	0.0004
ARCH (Alpha)	-0.010485	0.0067712	-1.548	0.1216
GARCH (Beta)	0.918227	0.012845	71.48	0.0000
GJR (Gamma)	0.156464	0.023806	6.572	0.0000
Asymmetry	-0.135466	0.024181	-5.602	0.0000
Tail	9.477067	1.6445	5.763	0.0000

No. Observations : 2823 No. Parameters : 7
 Mean (Y) : 0.00043 Variance (Y) : 0.00019
 Skewness (Y) : -0.04651 Kurtosis (Y) : 8.56106
 Log Likelihood : 8700.885

The sample mean of squared residuals was used to start recursion.
 The condition for existence of the second moment of the GJR is observed.
 This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.567322$ with this distribution.)

In this estimation, this sum equals 0.996508.

The condition for existence of the fourth moment of the GJR is not observed.

The constraint equals 1.03794 (should be < 1). => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :

0.000547; 0.018967; -0.010485; 0.918227; 0.156464; -0.135466; 9.477067

Elapsed Time : 1.326 seconds (or 0.0221 minutes).

Starting estimation process...

** GARCH(2) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: GJR (1, 1) model.
No regressor in the conditional variance
Student distribution, with 8.48224 degrees of freedom.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 8686.98
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst (M)	0.000791	0.00017756	4.454	0.0000
Cst (V) x 10 ⁴	0.018193	0.0053053	3.429	0.0006
ARCH (Alpha)	-0.008522	0.0074157	-1.149	0.2506
GARCH (Beta)	0.916924	0.012752	71.90	0.0000
GJR (Gamma)	0.153043	0.023356	6.552	0.0000
Student (DF)	8.482240	1.3397	6.331	0.0000

No. Observations :	2823	No. Parameters :	6
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8686.977		

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is $\alpha(1) + \beta(1) + k \gamma(1) < 1$ (with $k = 0.5$ with this distribution.)
In this estimation, this sum equals 0.984924.
The condition for existence of the fourth moment of the GJR is not observed.
The constraint equals 1.01092 (should be < 1) while the degree of freedom is 8.48224 (should be ≥ 5) => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
0.000791; 0.018193; -0.008522; 0.916924; 0.153043; 8.482240
Elapsed Time : 0.929 seconds (or 0.0154833 minutes).

Starting estimation process...

```
*****  
** GARCH(1) SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: APARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 8650.09

No. Observations :	2823	No. Parameters :	5
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8650.092		

Estimated Parameters Vector :

0.000596; 2.512068; 0.069835; 0.924304; 0.919890

Parameters Names

Cst(M) ; Cst(V) x 10⁴ ; ARCH(Alpha1) ; GARCH(Beta1)
; APARCH(Gamma1) ;

The tests are not reported since there is no convergence.

Elapsed Time : 0.383 seconds (or 0.00638333 minutes).

CondV [1 - 2823] saved to Returns since 2003 - Kopie.xls

VaR_in(0.05) [1 - 2823] saved to Returns since 2003 - Kopie.xls

Starting estimation process...

** GARCH(2) SPECIFICATIONS **

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: APARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 9.1998 degrees of freedom.
and asymmetry coefficient (log xi) -0.13915.

Strong convergence using numerical derivatives
Log-likelihood = 8704.83
Please wait : Computing the Std Errors ...

Robust Standard Errors (Sandwich formula)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.000507	0.00018336	2.765	0.0057
Cst(V) x 10 ⁴	2.325915	0.50884	4.571	0.0000
ARCH(Alpha)	0.071366	0.0088369	8.076	0.0000
GARCH(Beta)	0.926024	0.0093411	99.13	0.0000
APARCH(Gamma)	1.026513	0.10773	9.529	0.0000
Asymmetry	-0.139150	0.024709	-5.632	0.0000
Tail	9.199801	1.7489	5.260	0.0000
APARCH(Delta)	1.000000			

No. Observations :	2823	No. Parameters :	7
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8704.826		

The sample mean of squared residuals was used to start recursion.
The condition for existence of $E(\sigma^\Delta)$ and $E(|e^\Delta|)$ is observed.
The constraint equals 0.998388 and should be < 1.

Estimated Parameters Vector :
0.000507; 2.325915; 0.071366; 0.926024; 1.026513; -0.139150; 9.199806
Elapsed Time : 1.007 seconds (or 0.0167833 minutes).

Starting estimation process...

```
*****  
** GARCH(1) SPECIFICATIONS **  
*****
```

The dataset is: C:\Users\Felix\Dropbox\Online Drive\DBS\Master
Thesis\Data\Price and Return Data>Returns since 2003 - Kopie.xls
The estimation sample is: 2003-01-01 - 2013-12-31
The dependent variable is: CDAX
Mean Equation: ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation: APARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 8.3984 degrees of freedom.

No convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 8690.13

No. Observations :	2823	No. Parameters :	6
Mean (Y) :	0.00043	Variance (Y) :	0.00019
Skewness (Y) :	-0.04651	Kurtosis (Y) :	8.56106
Log Likelihood :	8690.125		

Estimated Parameters Vector :
0.000755; 2.197407; 0.072373; 0.925059; 0.985838; 8.398397

Parameters Names

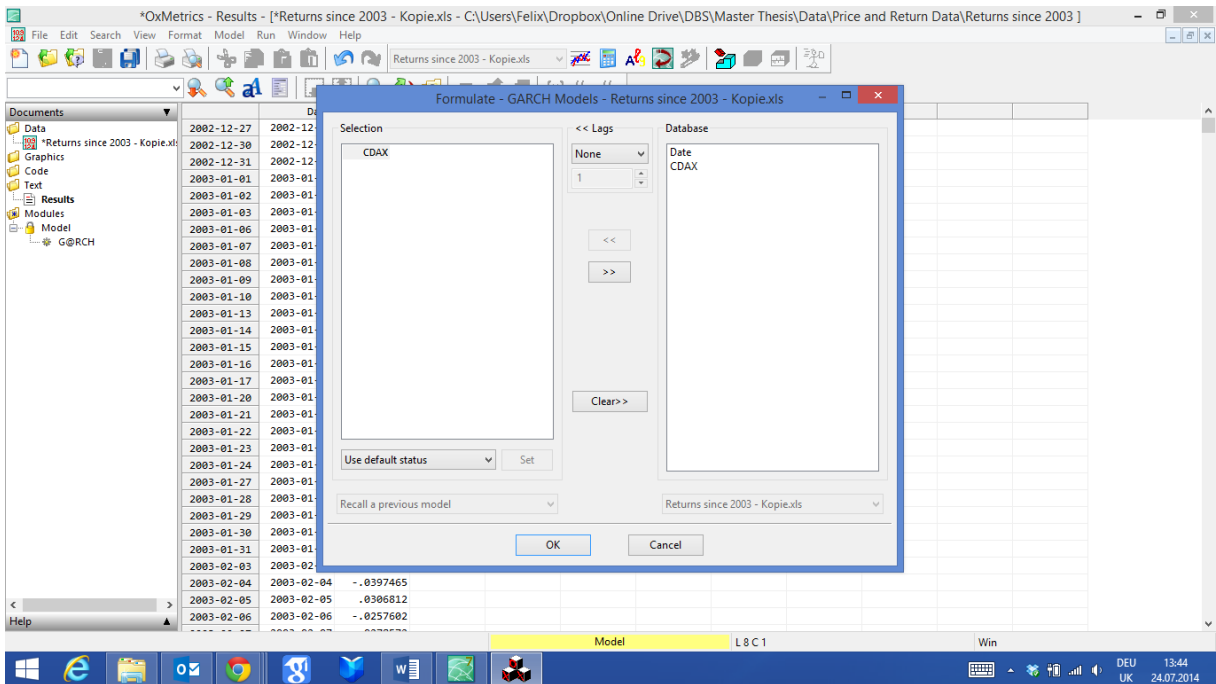
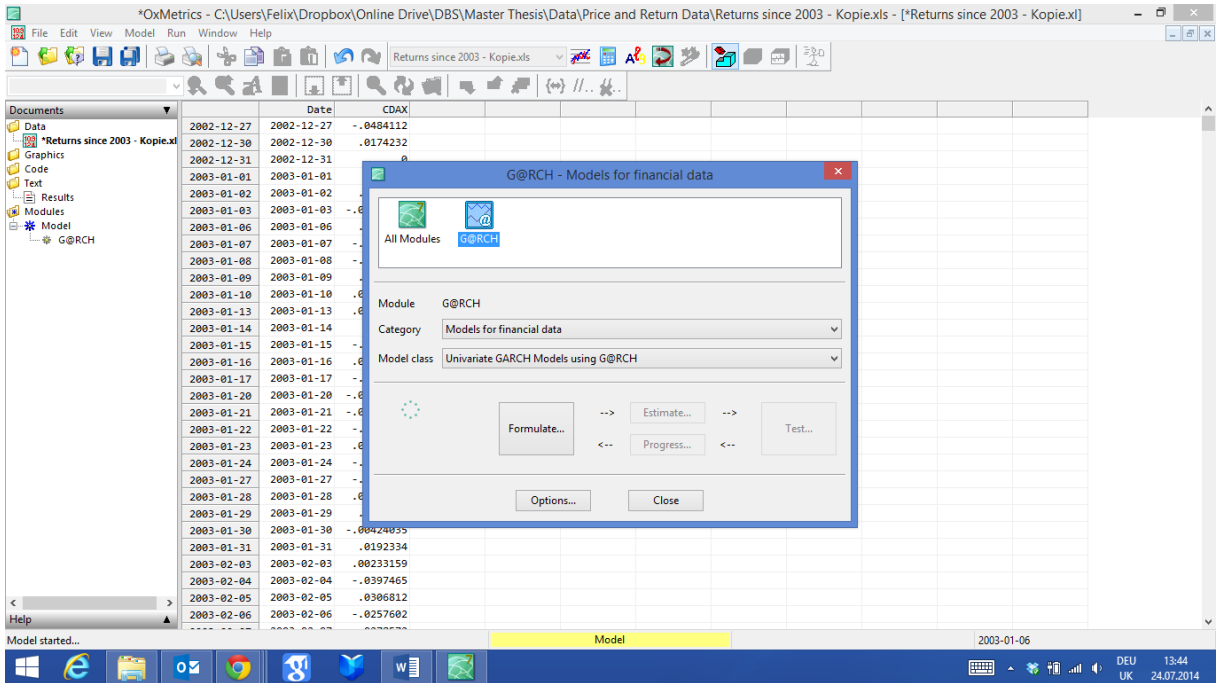
Cst(M) ; Cst(V) x 10⁴ ; ARCH(Alpha) ; GARCH(Beta1)
; APARCH(Gamma1) ; Student(DF) ;

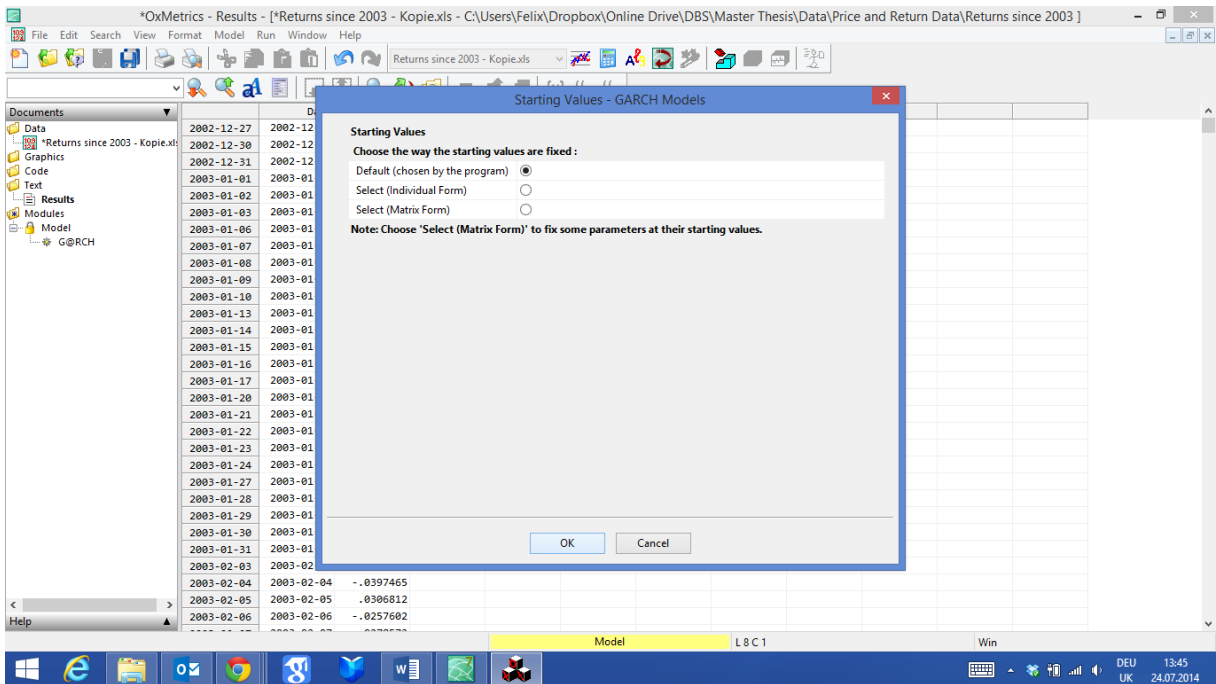
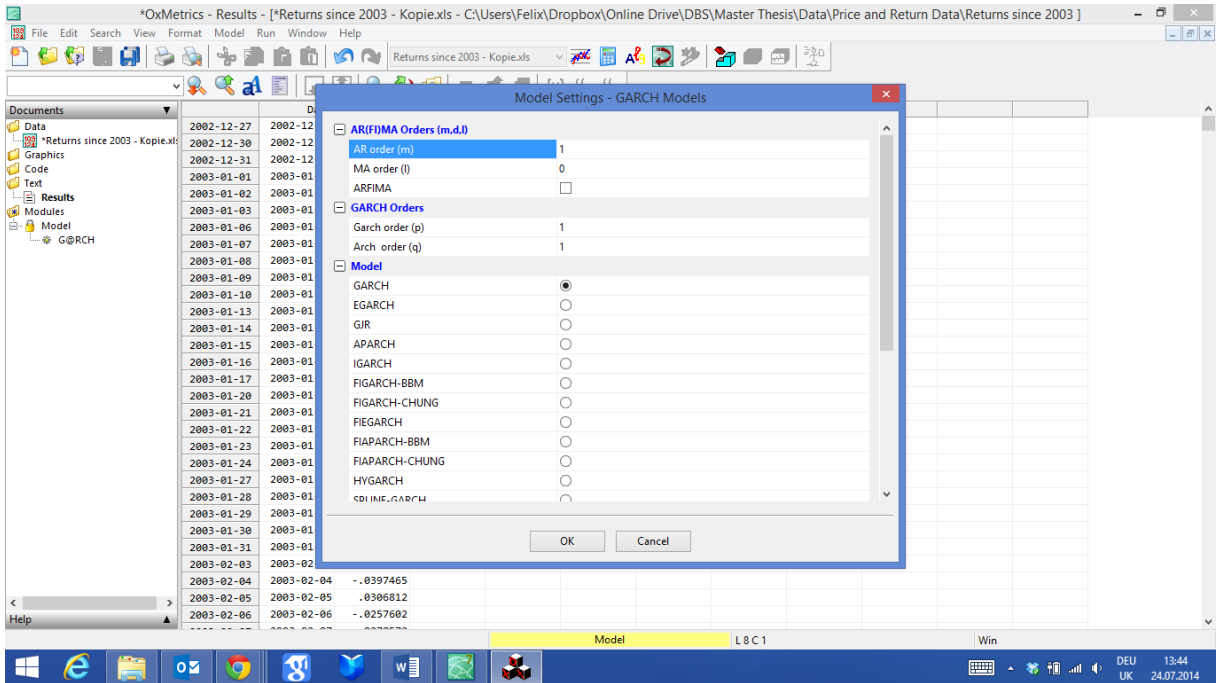
The tests are not reported since there is no convergence.
Elapsed Time : 0.603 seconds (or 0.01005 minutes).

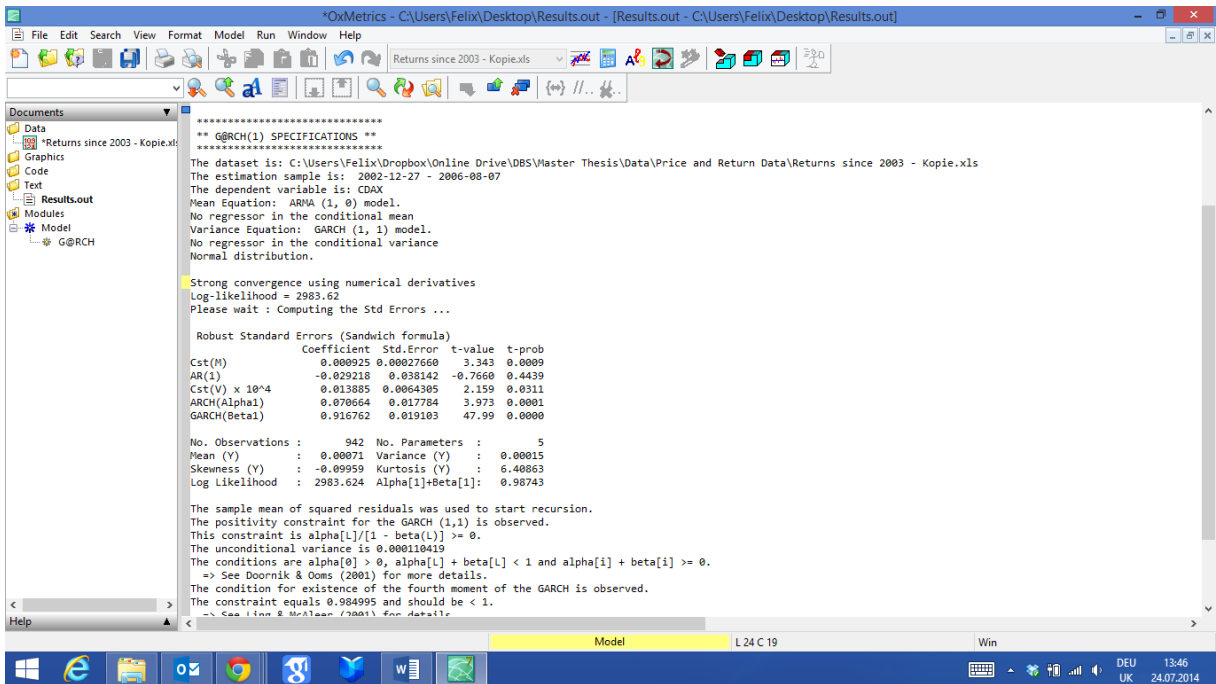
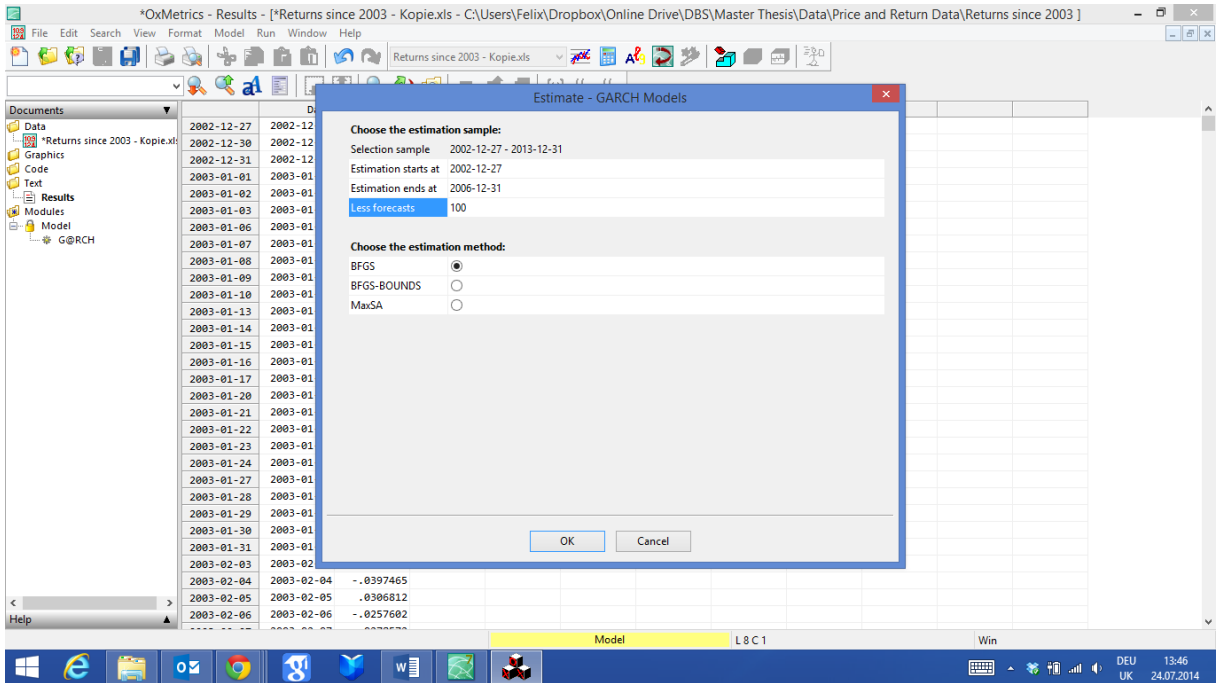
Appendix C Oxmetrics Screenshots

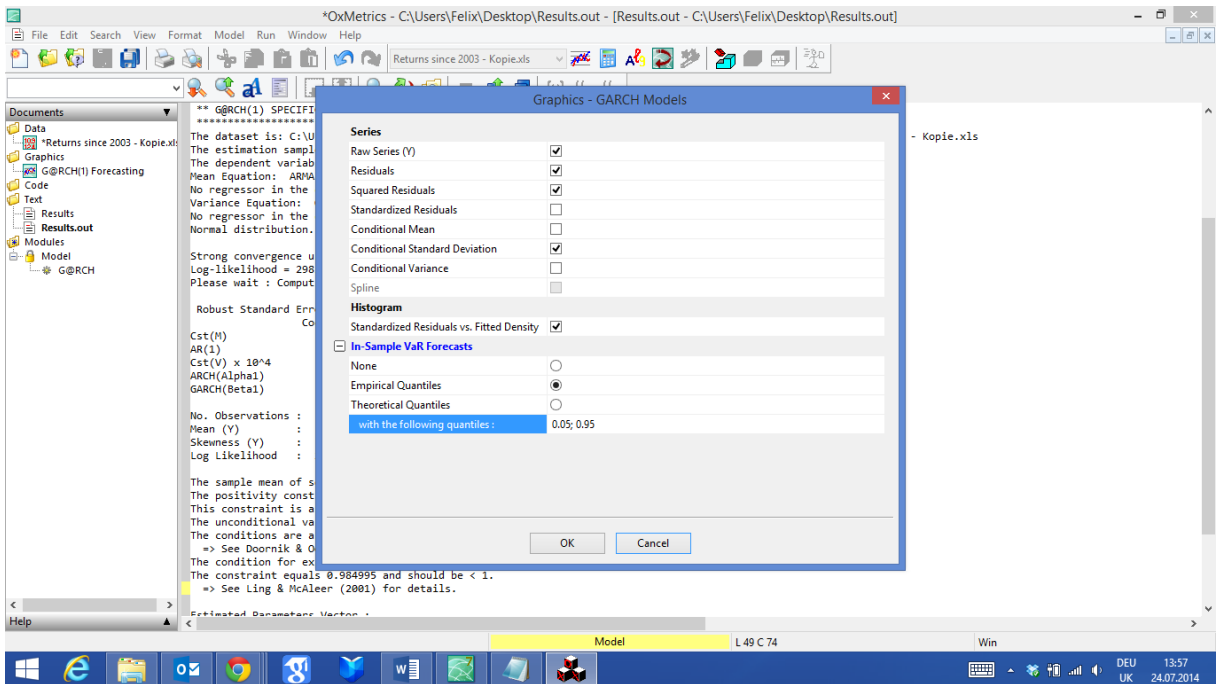
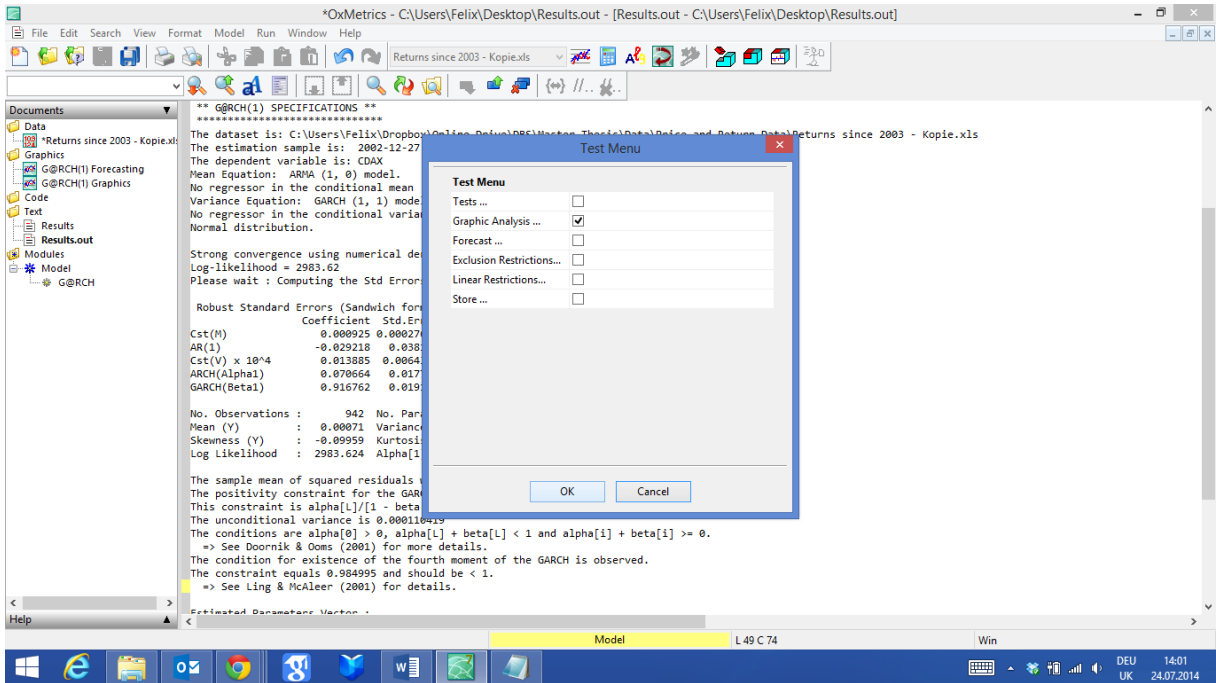
The screenshot shows an Excel spreadsheet with the following data:

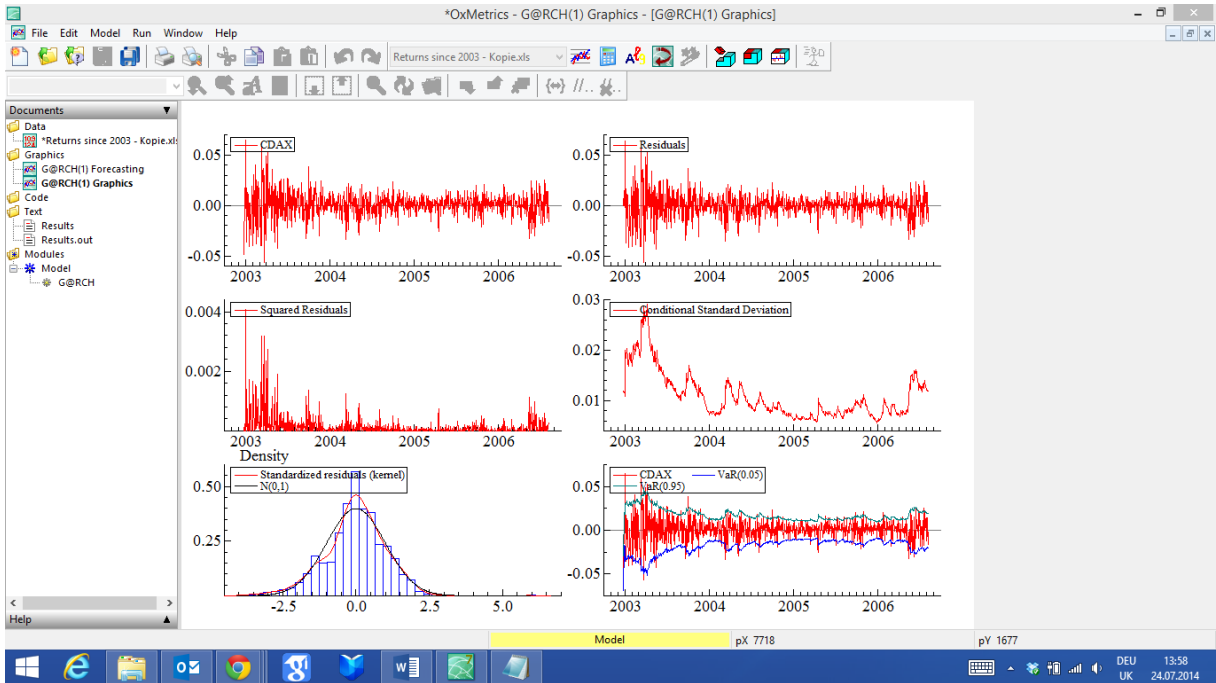
Date	CDAX
2002-12-27	-.0484112
2002-12-30	.0174232
2002-12-31	0
2003-01-01	0
2003-01-02	.0648277
2003-01-03	-.00318287
2003-01-06	.0175476
2003-01-07	-.0128331
2003-01-08	-.0360677
2003-01-09	.0133012
2003-01-10	.00152637
2003-01-13	.00729431
2003-01-14	.011217
2003-01-15	-.0143638
2003-01-16	.00231363
2003-01-17	-.0407888
2003-01-20	-.00728745
2003-01-21	-.00734095
2003-01-22	-.0223109
2003-01-23	.00224954
2003-01-24	-.0293938
2003-01-27	-.0251366
2003-01-28	.00765036
2003-01-29	.0117901
2003-01-30	-.00424035
2003-01-31	.0192334
2003-02-03	.00233159
2003-02-04	-.0397465
2003-02-05	.0306812
2003-02-06	-.0257602











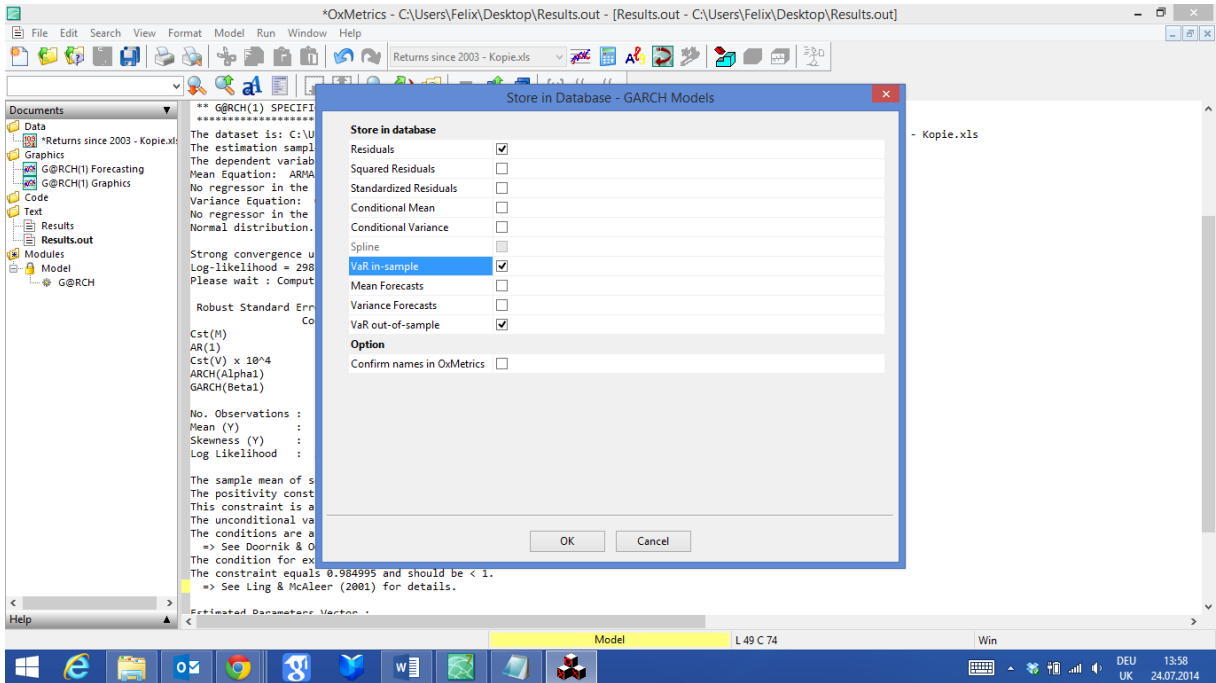
***OxMetrics - Results - [Results]**

Number of Forecasts: 100

Horizon	-0.05	0.05
1	-0.01873	0.02156
2	-0.0192	0.02183
3	-0.01916	0.02181
4	-0.01912	0.02097
5	-0.01909	0.02094
6	-0.01906	0.02091
7	-0.01903	0.02088
8	-0.019	0.02085
9	-0.01897	0.02082
10	-0.01894	0.02078
11	-0.01891	0.02075
12	-0.01888	0.02072
13	-0.01885	0.0207
14	-0.01882	0.02067
15	-0.01879	0.02064
16	-0.01876	0.02061
17	-0.01873	0.02058
18	-0.0187	0.02055
19	-0.01867	0.02052
20	-0.01865	0.0205
21	-0.01862	0.02047
22	-0.01859	0.02044
23	-0.01857	0.02042
24	-0.01854	0.02039
25	-0.01851	0.02036
26	-0.01849	0.02034
27	-0.01846	0.02031
28	-0.01844	0.02029
29	-0.01841	0.02026
30	-0.01839	0.02024
31	-0.01837	0.02021
32	-0.01834	0.02019
33	-0.01832	0.02017

Test Menu

- Tests ...
- Graphic Analysis ...
- Forecast ...
- Exclusion Restrictions ...
- Linear Restrictions ...
- Store ...



The screenshot shows the OxMetrics software interface displaying a data table. The table has the following columns: Date, CDAX, Res, VaR_out(0.05), VaR_out(0.95), VaR_in(0.05), and VaR_in(0.95). The data is as follows:

Date	CDAX	Res	VaR_out(0.05)	VaR_out(0.95)	VaR_in(0.05)	VaR_in(0.95)
2002-12-27	-0.0484112	0	missing	missing	-0.092899	-0.0299122
2002-12-30	0.174232	-0.015057	missing	missing	-0.0177308	0.201727
2002-12-31	0	-0.00442682	missing	missing	-0.201392	0.186788
2003-01-01	0	-0.00951752	missing	missing	-0.0188637	0.185087
2003-01-02	0.0648277	0.0638759	missing	missing	-0.018138	0.178657
2003-01-03	-0.00318287	-0.00224049	missing	missing	-0.035897	0.300283
2003-01-06	0.175476	0.0165028	missing	missing	-0.0325032	0.307691
2003-01-07	-0.0128331	-0.0132721	missing	missing	-0.0326519	0.297584
2003-01-08	-0.0360677	-0.0373944	missing	missing	-0.0310192	0.29986
2003-01-09	0.0139012	0.0112957	missing	missing	-0.0335783	0.335338
2003-01-10	0.00152637	0.00963251	missing	missing	-0.0339723	0.311624
2003-01-13	0.00729431	0.00638716	missing	missing	-0.032227	0.302649
2003-01-14	0.011217	0.0104784	missing	missing	-0.0311922	0.298301
2003-01-15	-0.0143638	-0.0149878	missing	missing	-0.030404	0.281156
2003-01-16	0.00231363	0.0099422	missing	missing	-0.0292145	0.284713
2003-01-17	-0.0407888	-0.041673	missing	missing	-0.028477	0.268988
2003-01-20	-0.00728745	-0.00943096	missing	missing	-0.0320707	0.324582
2003-01-21	-0.00734095	-0.00850562	missing	missing	-0.0319516	0.305065
2003-01-22	-0.0223109	-0.0234771	missing	missing	-0.0308544	0.295373
2003-01-23	0.00224954	0.00645908	missing	missing	-0.0310089	0.304991
2003-01-24	-0.0293938	-0.0302798	missing	missing	-0.0304092	0.286144
2003-01-27	-0.0251366	-0.0269472	missing	missing	-0.0313656	0.312055
2003-01-28	0.00765036	0.00596417	missing	missing	-0.0325268	0.319998
2003-01-29	0.0117901	0.0110619	missing	missing	-0.0322119	0.29914
2003-01-30	-0.00424035	-0.00484762	missing	missing	-0.031416	0.289806
2003-01-31	-0.0192334	-0.0181577	missing	missing	-0.0297378	0.283771
2003-02-03	0.00233159	0.00194179	missing	missing	-0.0303687	0.276426
2003-02-04	-0.0397465	-0.0406301	missing	missing	-0.0286529	0.270537
2003-02-05	0.0306812	0.0285681	missing	missing	-0.0319677	0.323094

